Bayesian Augmented ACD Models in Analysis of Financial Trade Durations: Evidence from the Warsaw Stock Exchange

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Abstract

In recent years, very popular in modelling the durations between the selected events of the transaction process (trade durations or price durations) and modelling of financial market microstructure effects have become autoregressive conditional duration models (the ACD models) introduced by Engle and Russell (1998). The aim of the paper is to develop Bayesian inference for some augmented specifications of the ACD models. Different specifications of ACD models will be considered and compared with particular emphasis on Box-Cox ACD model, augmented Box-Cox ACD model and augmented (Hentschel) ACD model. The models with the Burr distribution and the generalized Gamma distribution for innovation term will be considered in the analysis. Bayesian inference will be presented and practically used to estimation and prediction of augmented ACD models describing financial durations. The MCMC methods including Metropolis-Hastings algorithm are suitably adopted to obtain posterior densities of interest as well as marginal data densities. The empirical part of work will include modelling of trade durations of selected equities from the Warsaw Stock Exchange.

JEL Classifications: C50, C58, C11, C22

Keywords: market microstructure effects, autoregressive conditional duration models, trade durations, Bayesian inference

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1. Introduction

Over the past fifteen years, there has been a considerable amount of empirical research in financial market microstructure. This research has focused both on theoretical and on empirical models, on price driven and order driven markets, on the importance of information asymmetry between traders, on liquidity, on the role of information in trading process and on the interaction between participants of the markets. New perspectives for the empirical studies and analysis of financial market microstructure processes and effects have been made possible by the availability of financial data recorded on high-frequency level. The accessibility of financial transactions data has inspired a research area which over the last fifteen years emerged to one of the major areas in econometrics. The growing popularity of high-frequency econometrics is caused by technological progress in trading systems and trade recording as well as an increasing importance of intraday trading, optimal trade execution and liquidity dynamics. A major reason for the great academic (and also practical) interest in high-frequency finance is that market structures and the process of trading are subject to changes. Transaction data, provided by various exchanges and trading platforms, make strong academic interest as they allow to analyze the impact of institutional settings on the trading process, price discovery as well as the market outcome and enable to study market dynamics and behavior of traders on the lowest possible aggregation level (see Hautsch (2012)). Monitoring asset prices as well as liquidity supply and demand on a maximally high observation frequency opens up possibilities to construct more efficient estimators, measures and predictors for price volatility and liquidity risks. Therefore, these issues require the development of econometric tools and models which are able to cope with specific data and tasks.

The financial ultra-high frequency data (UHF data) called transaction data or tick-by-tick data is defined to be a full record of transactions and their associated characteristics. The ultra-high frequency transaction data contain two types of observations. One is the time of transaction. The other is a vector of the quantities (the marks), observed at the time of transaction (price, volume). A key property of transaction data is the irregular spacing in time. The time intervals between subsequent transactions called trade durations can bring important content about the intensity of the information flow to the market. Moreover, the market microstructure papers by Diamond and Verrecchia (1987), Glosten and Milgrom (1985), Hasbrouck (1988) , O'Hara (1995) and others emphasize that the waiting times between events of trading process such as trades, quote updates, price changes and order arrivals play a key role in understanding the processing of private and public information in financial markets. Hence, duration analyses may furnish information on the microstructure of the financial market, affording a more accurate insight into various market interdependencies. By using the tick-by-tick data it is possible to define almost any event of interest, and the corresponding duration sequence, not only sequence of trade durations. In recent years, very popular in modelling the durations between the selected events of the transaction process (trade durations or price durations) and modelling of financial market microstructure effects received autoregressive conditional duration models (the ACD models) introduced by Engle and Russell (1998).

The aim of the paper is to present Bayesian inference to some augmented specifications of the ACD models. Different specifications of ACD models will be considered and compared with particular emphasis on augmented Box-Cox ACD model and Hentschel ACD model. The Burr distribution and the generalized Gamma distribution of innovation term will be considered in the analysis. Bayesian inference will be presented and practically used to estimation and prediction of augmented ACD models describing financial durations. The empirical part of work will include modelling of trade durations of selected equities from the Warsaw Stock Exchange.

2. The ACD approach for durations

Let $t_1, t_2, \ldots, t_n, \ldots$ be a sequence of the arrival times of the point process, which are represented by the times of the financial events such as trades, quotes, etc. One common way of analyzing financial point process, point process that consist of arrival times of events linked to the trading process, is by modeling the process of durations between consecutive points. Let $x_i = t_i - t_{i-1}$ denote the duration, time spell between two events occurring at times t_{i-1} and t_i. Let Ψ_i denote the expectation of the duration conditional on all information available at time t_{i-1}. The idea behind the ACD model is a dynamic parameterization of the conditional duration mean:

$$\Psi_i = E(x_i \mid \Im_{i-1}; \theta) = E(x_i \mid x_{i-1}, \ldots, x_1; \theta) = \Psi_i(x_{i-1}, \ldots, x_1; \theta),$$

where θ denotes a vector of parameters and $\Im_{i-1}$ denotes the information set available at time t_{i-1}. It is also assumed that the durations x_i are given by

$$x_i = \Psi_i \cdot \varepsilon_i \,,$$

where ε_i are the standardized durations and follow an i.i.d process defined on positive support with density function $f_\varepsilon(\varepsilon_i)$ for ε_i and $E(\varepsilon_i)=1$, see Engle and Russell (1998), Hautsch (2004), Hautsch (2012). Different types of ACD models can be divided either by the choice of the functional form for the conditional mean function Ψ_i (see Bauwens, Giot (2000), Hautsch (2001), Hautsch (2004), Fernandes and Grammig (2006)) or by the choice of the distribution for ε_i (see Lunde (1999), Grammig and Maurer (2000)).

The basic ACD specification proposed by Engle and Russell (1998) relies on a linear parameterization of the conditional mean function

$$\Psi_i = \omega + \sum_{j=1}^{p} \alpha_j x_{i-j} + \sum_{j=1}^{q} \beta_j \Psi_{i-j} \,,$$

where $\omega > 0, \alpha_j \geq 0, \beta_j \geq 0$ are a sufficient but not necessary conditions for positivity of the process. This model is referred to ACD(p,q) model. Indeed, conditional duration is a function of previous observations and the previous conditional durations.

The ACD model is widely viewed as the counterpart of the GARCH model for the time intervals between the individual transactions (trade duration data). Autoregressive specification of the ACD model allows for capturing the duration clustering between events of transaction process, ie. short (long) durations are followed by other short (long) durations, in a way similar to the GARCH model accounting for the volatility clustering, see Pacurar (2008). The attractiveness of the basic ACD models result from their simple specifications and, at the same time, the possibility of converting them into a popular econometric time series analysis tools, such as ARMA models and GARCH models. The ACD models are considered, moreover, as models of duration, for easy to estimation. In addition, they allow straightforward computation of the forecasts (in literature only point one-step-ahead forecasts are considered!). It should be noted that the basic specification of the ACD models may be too restrictive to properly describe financial duration processes. This gave rise to numerous extensions of the ACD models. Extensions of the ACD models are both associated with the improvement of the specification of the conditional duration equation (see Bauwens, Giot (2000), Hautsch (2001), Hautsch (2004), Fernandes and Grammig (2006)), and the use of different random distributions (see Lunde (1999), Grammig and Maurer (2000)).

3. Some remarks about ML estimation and proposition of Bayesian inference

The statistical properties of the basic ACD(1,1) model are well investigated, see Engle and Russell (1998), Bauwens and Giot (2000). However, the properties of extensions of the basic ACD model are not well-known. In the case of ACD models the classical approach to inference about the parameters of models is based on the estimation by quasi-maximum likelihood method (QML method) or by maximum likelihood method (ML method). Following the similarity between the ACD and the GARCH model, the results of Lee and Hansen (1994) and Lumsdaine (1996) on the QMLE properties for the GARCH(1,1) model are formalized for the EACD(1,1) model (the ACD model with the standard exponential distribution for ε_i) by Engle and Russell (1998). Under the conditions of their theorem, consistent and asymptotically normal QML estimates of parameters are obtained, even if the distribution of ε_i is not exponential. Moreover, a crucial and binding assumption for obtaining QML consistent estimates of the ACD model is that the conditional expectation of durations is correctly specified, see Pacurar (2008). In addition, it has to be taken into account that these results are based on the linear EACD(1,1) model and cannot necessarily be carried over and extended to more general cases, like nonlinear models. The QML estimation provides consistent estimates, but this comes at the cost of efficiency. In practice, fully efficient maximum likelihood estimates might be preferred, see Pacurar (2008). On the other hand, QML parameter estimates can be biased in finite samples. It is shown in Grammig and Maurer (2000). The authors perform Monte Carlo simulations and show that the QML estimation of the models may perform poorly in finite samples, even in large samples such as 10000 – 15000 observations. As we see, the ML method does not guarantee in all situations that estimators are unbiased, efficient and consistent, see Grammig and Maurer (2000), Hautsch (2012).

Thus, due to problems concerning the basic properties of the ML estimator in the case of ACD models, Bayesian inference seems to be not only a competitive approach, but also more formally justified. It is worth noting that in recent years, in addition to the dynamic development of financial econometrics and analysis of high-frequency data, it can be observed increased interest in Bayesian inference in empirical sciences (see for example Osiewalski and Pipień (2004a), (2004b)). Bayes theorem provides a convenient and versatile way of inference about the unknown parameters and latent variables. Bayesian methodology provides inference tools to describe the uncertainty in strict probabilistic and non-asymptotic way. Strong interest in financial UHF data and the growing interest in ACD models and their use in modeling financial time series makes an attempt to present the Bayesian approach. Although in the literature, the ACD models are already established, these researches will be the first study on Bayesian inference for the whole class of the ACD models. To the author's best knowledge such a Bayesian study has not appeared yet in the econometrics literature. This study will provide a creative contribution to the development of research on Bayesian ACD models. Moreover, application of the ACD models in the modeling of trading activity of stocks on the Polish market seems to be interesting and useful.

4. Specification of the ACD models considered in the paper

Different specifications of ACD models will be taken into consideration. The class of the ACD models contains various extensions of the basic linear ACD model allowing for additive as well as multiplicative stochastic components. Moreover, it contains parameterizations capturing not only linear but also more flexible news impact curves. In our empirical researches, the following specifications for the conditional mean function Ψ_i are considered:

- a linear ACD model – ACD(1,1), see Engle and Russell (1998):

  $$\Psi_i = \omega + \alpha \cdot x_{i-1} + \beta \cdot \Psi_{i-1},$$

  where: $\omega > 0, \alpha \geq 0, \beta \geq 0, \alpha + \beta < 1$,

- a logarithmic ACD model – LACD(1,1) that ensures the non-negativity of durations without any parameter restrictions, see Bauwens and Giot (2000), Lunde (2000):

  LACD₁
  $$\begin{aligned}\ln \Psi_i &= \omega + \alpha \cdot \ln \varepsilon_{i-1} + \beta \cdot \ln \Psi_{i-1} \\ &= \omega + \alpha \cdot \ln x_{i-1} + (\beta - \alpha) \cdot \ln \Psi_{i-1},\end{aligned}$$

  where $|\beta| < 1$,

- a Box-Cox ACD model – BCACD(1,1), see Hautsch (2001):

  $$\Psi_i^{\delta_1} = \omega + \alpha \cdot \varepsilon_{i-1}^{\delta_2} + \beta \cdot \Psi_{i-1}^{\delta_1},$$

  where: $\omega > 0, \alpha \geq 0, 0 \leq \beta < 1, \delta_1 > 0, \delta_2 > 0$,

- an asymmetric logarithmic ACD model – AsLACD(1,1), see Fernandes and Grammig (2006):

$$\ln\Psi_i = \omega + \alpha\cdot[|\,\varepsilon_{i-1} - b\,| + c\cdot(\varepsilon_{i-1} - b)] + \beta\cdot\ln\Psi_{i-1},$$

where: $b > 0$ and $|\beta| < 1$,

- an augmented Box-Cox ACD model – ABCACD(1,1), see Hautsch (2004), (2012):

$$\Psi_i^{\delta_1} = \omega + \alpha\cdot[|\,\varepsilon_{i-1} - b\,| + c\cdot(\varepsilon_{i-1} - b)]^{\delta_2} + \beta\cdot\Psi_{i-1}^{\delta_1},$$

where: $\omega > 0, \alpha \geq 0, 0 \leq \beta < 1, \delta_1 > 0, \delta_2 > 0, b \geq 0, |c| \leq 1$,

- a Hentschel ACD model – HACD(1,1), see Fernandes and Grammig (2006):

$$\Psi_i^{\delta_1} = \omega + \alpha\cdot\Psi_{i-1}^{\delta_1}\cdot[|\,\varepsilon_{i-1} - b\,| + c\cdot(\varepsilon_{i-1} - b)]^{\delta_2} + \beta\cdot\Psi_{i-1}^{\delta_1},$$

where: $\omega > 0, \alpha \geq 0, 0 \leq \beta < 1, \delta_1 > 0, \delta_2 > 0, b \geq 0, |c| \leq 1$.

The standard ACD model assumes that the distribution of innovation ε_i is an exponential distribution with mean equal to 1. I consider the ACD specifications with more flexible innovation distributions: the Burr distribution and the generalized Gamma distribution. For a Burr distribution for innovations ε_i, the conditional distribution of duration x_i required to calculate the likelihood function can be written as

$$f(x_i \mid \Im_{i-1};\theta) = \frac{\kappa}{x_i}\cdot\left(\frac{x_i\cdot\mu}{\Psi_i}\right)^{\kappa}\cdot\left[1 + \sigma^2\cdot\left(\frac{x_i\cdot\mu}{\Psi_i}\right)^{\kappa}\right]^{-1-\frac{1}{\sigma^2}},$$

where $\mu = \dfrac{\Gamma\left(1+\frac{1}{\kappa}\right)\cdot\Gamma\left(\frac{1}{\sigma^2}-\frac{1}{\kappa}\right)}{\left(\sigma^2\right)^{1+\frac{1}{\kappa}}\cdot\Gamma\left(1+\frac{1}{\kappa}\right)}$ and $0 < \sigma^2 < \kappa$. On the other hand for a generalized Gamma distribution for innovations ε_i, the conditional distribution of duration x_i can be written as

$$f(x_i \mid \Im_{i-1};\theta) = \frac{\gamma}{x_i\cdot\Gamma\left(\frac{p}{\gamma}\right)}\cdot\left(\frac{x_i\cdot\mu}{\psi_i}\right)^{p}\cdot\exp\left[-\left(\frac{x_i\cdot\mu}{\psi_i}\right)^{\gamma}\right],$$

where $\mu = \dfrac{\Gamma\left(\frac{1+p}{\gamma}\right)}{\Gamma\left(\frac{p}{\gamma}\right)}$ and $p, \gamma > 0$.

5. Bayesian inference for the ACD models

All ACD models are estimated in the Bayesian framework. The aim of research is to obtain the posterior distribution. Obviously, the posterior distribution is proportional to the product of the likelihood and the prior density. In case of the ACD models which are taken into consideration, the posterior and the conditional posterior distributions have non-standard distributions, so it is impossible to use Gibbs sampling algorithm. Instead, the random walk Metropolis-Hastings algorithm is adopted to simulate from the posterior. As a proposal density, the multivariate Student-t distribution with three degrees of freedom and a fixed covariance matrix was used.

The Bayesian methodology allows for the comparison and ranking of the models by Bayes factor or posterior odds. The Bayes factor for model M_i versus M_j is defined as $B_{ij} = p(x \mid M_i) / p(x \mid M_j)$ which is the ratio of the marginal densities of the data of these models. For calculation of the marginal data density $p(x \mid M_i)$ for model M_i we use the well-known method of Newton and Raftery (1994) adapted by many authors. Estimator of Newton and Raftery is a harmonic mean:

$$\hat{p}(x \mid M_i) \approx \left(\frac{1}{N} \sum_{q=1}^{N} \frac{1}{p_i(x \mid \theta_{(i)}{}^{(q)})} \right)^{-1},$$

where $p_i(x \mid \theta_{(i)}{}^{(q)})$ denotes the likelihood for model M_i, N denotes the number of MCMC draws and $\theta_{(i)}{}^{(q)}$ is a q-th MCMC draw from the posterior.

6. Empirical results - preview

The empirical verification of the models presented above is carried out on the basis of time series involving trades in the shares of three companies listed in the main Polish index of the Warsaw Stock Exchange, the WIG20 index: media company Agora (AGORA), Polish Telecom (TPSA) and Polish bank PKOBP S.A. (PKOBP). The companies are chosen randomly but in a such way that they differ with respect to their stocks liquidity, their trading intensity. The analysis covers transactions closed during the continuous quotation phase. On the basis of such time series, durations between each transaction were determined. Durations are measured with an accuracy of 1 second. Additionally, the time lags between the close of the session and the opening of next day’s trading were removed or assumed to be equal to zero (this solution is frequently applied).

I find some common properties that Polish market shares with foreign, developed exchanges. Polish trade durations display strong autocorrelation and overdispersion. As documented by Engle and Russell (1998), Bauwens and Giot (2000) and others, trade durations feature a strong time-of-day effect. Diurnal patterns of Polish trade durations are similar. The durations between transactions are markedly shorter after the opening and before the close of the session than at midday. The extent of trading activity between 12:00 a.m. and 2:00 p.m. is noticeably less, due, amongst others, to the lunchtime effect. Therefore, I consider diurnally adjusted durations obtained by taking the ratios of the plain durations and diurnal factor. The intraday seasonality patterns are estimated as the Nadaraya-Watson estimator of regression of the duration on the time of the day, determined separately for each day of the week.

My results of Bayesian inference are based on 2,000,000 states of the Markov chain, generated after 200,000 burnt-in states. Applications of MCMC simulations require monitoring convergence of the chain to its limiting stationary distribution, see Cowles and Carlin (1996), Osiewalski and Pipień (2004a). In my research it is used a simple tool that was proposed by Yu and Mykland (1994) namely CUMSUM statistics for the Monte Carlo estimates of the posterior means of individual parameters.

The joint prior density of the parameter vector is a product of the prior densities of its components. The prior densities for the parameters of the conditional mean functions Ψ_i are independent normal N(0;25) or, if necessary, are set to be truncated normal N(0;25). The priors for the Burr distribution parameters κ and σ^2 are set to be truncated normal N(0;25) with positive supports and the restriction $\sigma^2 < \kappa$. The priors for the generalized Gamma distribution parameters γ and p are set to be truncated normal N(0;25) or N(0;100) with positive supports, respectively.

Now I comment on some empirical results. Full empirical result will be presented during conference. Empirical analysis for Polish data showed that the best model for all equities (in light of the posterior probabilities) is the Box-Cox ACD model with a generalized Gamma distribution for innovations. The Lindley's tests exhibit that all the coefficients are significant. The posterior means of parameter β are large and around 0.9-0.95 (it depends on equity). The posterior means of parameter α are rather small and around 0.05-0.10, so the effect of the last standardized duration ε_{i-1} is small. The Lindley's tests exhibit also that all estimated parameters of the generalized gamma distribution are significant. The posterior means of parameter γ are greater than 0 and smaller than 1. The posterior means of parameter p are greater than 1. All estimated parameters of the Burr distribution are also significant. The posterior means of parameter κ are greater than 1 and the posterior means of parameter σ^2 are greater than 0. This exhibits that the generalized gamma distribution and the Burr distribution are much more appropriate for duration innovations than the exponential distribution. It must be said that the ACD models with the generalized gamma distribution for innovations definitely outperformed the ACD models with the Burr distribution for innovations.

In ranking of the models the asymmetric logarithmic ACD model is on the second place. The Box-Cox ACD model with a generalized Gamma distribution is about 3 orders of magnitude more probable a posteriori than the asymmetric logarithmic ACD model. Needless to say, there are some problems with convergence of Markov chain in augmented Box-Cox ACD model and Hentschel ACD model. There are problems with identification of parameters b and c in both specifications. Therefore, the Box-Cox transformation in Box-Cox ACD model and the asymmetric response to shocks in the asymmetric logarithmic ACD model indeed work but it seems, to some extent, as a substitutes, do not work together like in augmented Box-Cox ACD model and Hentschel ACD model.

7. Conclusions

Empirical analysis for Polish data showed that the best model for all equities (in light of the posterior probabilities) is the Box-Cox ACD model with a generalized Gamma distribution for innovations. There are some problems with convergence of Markov chain in augmented Box-Cox ACD model and Hentschel ACD model. Especially, there are problems with convergence of Markov chain for parameters b and c in both specifications. Moreover, the Box-Cox transformation and the asymmetric response to shocks indeed work but it seems, to some extent, as a substitutes. It seems that the Box-Cox transformation is sufficient and introduction to model the effects of asymmetry for the error term (augmented Box-Cox ACD model and Hentschel ACD model) seems unnecessary.

The ACD models with the generalized gamma distribution for innovation definitely outperformed the ACD models with the Burr distribution for innovation. The generalized gamma distribution turned out to be much more flexible than the Burr distribution. Thanks to the Bayesian inference there were obtained the predictive distributions of time of apperance of k consecutive transactions (in the literature only talked about one-step-ahead forecasts).

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