Efficient Bayesian Inference for Multivariate Factor Stochastic Volatility (SV) Models

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Multivariate factor stochastic volatility models are increasingly used for the analysis of multivariate financial and economic time series because they can capture the volatility dynamics by a small number of latent factors. The main advantage of such a model is its parsimony, since all variances and covariances of a vector of time series are governed by a low-dimensional common factor with the components following independent SV models. The model reads

$$\mathbf{y}_t = \mathbf{\Lambda}\mathbf{f}_t + \mathbf{\Sigma}_t^{1/2}\boldsymbol{\epsilon}_t, \qquad \mathbf{\Sigma}_t = \text{Diag}(\exp(h_{1t}), \ldots, \exp(h_{mt})), \qquad \boldsymbol{\epsilon}_t \sim N_m(\mathbf{0}, \mathbf{I}_m), \quad (1)$$

$$\mathbf{f}_t = \mathbf{V}_t^{1/2}\mathbf{u}, \qquad \mathbf{V}_t = \text{Diag}(\exp(h_{m+1,t}), \ldots, \exp(h_{m+r,t})), \qquad \mathbf{u} \sim N_r(\mathbf{0}, \mathbf{I}_r), \quad (2)$$

where $\mathbf{y}_t = (y_{1t}, \ldots, y_{mt})'$, $t = 1, \ldots, T$ and $\mathbf{\Lambda}$ is an unknown $m \times r$ factor loading matrix with elements Λ_{ij}. The standard assumption is that $\mathbf{f}_t$, $\mathbf{f}_s$, $\boldsymbol{\epsilon}_t$, and $\boldsymbol{\epsilon}_s$ are pairwise independent for all t and s. Both the latent factors and the idiosyncratic shocks are allowed to follow different stochastic volatility processes, i.e.

$$h_{it} = \mu_i + \phi_i(h_{i,t-1} - \mu_i) + \sigma_i\eta_{it}, \qquad \eta_{it} \sim N(0, 1). \quad (3)$$

Multivariate factor SV models have recently been applied to important problems in financial econometrics such as asset allocation (Aguilar & West, 2000) and asset pricing (Nardari & Scruggs, 2007). They extend standard factor pricing models such as the arbitrage pricing theory and the capital asset pricing model. However, as opposed to SV factor models, standard factor pricing models do not attempt to model the dynamics of the volatilities of the asset returns and usually assume that the covariance matrix $\mathbf{\Sigma}_t \equiv \mathbf{\Sigma}$ is constant. Empirical evidence suggests that multivariate factor SV models are a promising approach for modeling multivariate time-varying volatility, explaining excess asset returns, and generating optimal portfolio strategies.

For high dimensional problems of this kind, Bayesian MCMC estimation is a very efficient estimation method, however, it is associated with a considerable computational burden, when the number of assets is moderate to large. Several papers (Kim et al., 1998; Chib et al., 2002, 2006; Omori et al., 2007) consider a variety of multivariate stochastic volatility (MSV) models with error distributions arising from Gaussian or Student-t distributions that allow for both symmetric and asymmetric conditional distributions. In the multivariate case, the correlation between variables is governed by several latent factors.

Bayesian estimation relies on data augmentation: we introduce the latent volatilities $\mathbf{h} = (\mathbf{h}_i)$, where $\mathbf{h}_i = (h_{i0}, \ldots, h_{iT})$ for $i = 1, \ldots, m + r$, and the latent factors $\mathbf{f} = (\mathbf{f}_1 \cdots \mathbf{f}_T)$ as latent data. Given $\mathbf{h}$, (1) is a standard factor model and $\mathbf{f}$ and $\mathbf{\Lambda}$ may be sampled as in Lopes & West (2004). Given $\mathbf{f}$, (1) can be easily transformed into $m+r$ independent univariate SV models where the latent equation (3) is combined for $i = 1, \ldots, m$ with the observation equation $\log(y_{it} - \mathbf{\Lambda}_{i\cdot}\mathbf{f}_t)^2 = h_{it} + \log(\epsilon_{it}^2)$, and for $i = m+1, \ldots, m+r$ with the observation equation $\log(f_{i-m,t})^2 = h_{it} + \log(\eta^2_{i-m,t})$. Hence Bayesian estimation method for a multivariate factor SV models depends on how efficiently a univariate SV model is estimated. We use the popular approach of auxiliary mixture sampling (Kim et al., 1998; Chib et al., 2002, 2006; Omori et al., 2007) and approximate the distribution of $\log(\epsilon^2_{it})$ by a mixture of 10 normal distributions. Conditional on the component indicator r_{it}, this leads to a Gaussian linear state space model. Rather than using standard forward-filtering backward-sampling (Frühwirth-Schnatter, 1994; Carter & Kohn, 1994) to draw the volatilities, we apply a sparse Cholesky factor algorithm (see e.g. McCausland et al., 2011) to sample "all without a loop" (AWOL) from the high-dimensional joint density of all volatilities. This reduces computing time considerably, as it allows joint sampling without running a filter. Also, we consider various reparameterizations of the augmented SV model. Under the standard parameterization, augmented MCMC estimation turns out to be inefficient, especially if the volatility of volatility parameter σ_i in the latent state equation is small. By considering a non-centered version of the SV model, this parameter is moved to the observation equation. Using MCMC estimation for this transformed model reduces the inefficiency factor in particular for the volatility of volatility parameter considerably. Finally, we adopt an ancillarity-sufficiency interweaving strategy (Yu & Meng, 2011) outperforming both centered and non-centered parameterizations in terms of sampling efficiency with respect to all parameters.

To show the effectiveness of our approach and its suitability for real world applications, we apply the model to a vector of daily exchange rate data.

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