Bayesian analysis of polynomial weak form reduced rank structure in VEC models
DRAFT

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Abstract

The main goal of the paper is the Bayesian analysis of the weak form of polynomial serial correlation common features.
After the introduction and discussion of the model, the methods will be illustrated by an empirical investigation of the price-wage nexus in the Polish economy.

Key Words: cointegration; Bayesian analysis; polynomial common cyclical features

JEL Classification: C11; C32; C53

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1 Introduction

While modelling time series we try to "catch" their most important features such as trends, serial correlation, seasonality etc., and while analysing a group of series we try to include in the model the most important properties of that series and as they are modelled together to find such features which are common. Following the idea of Engle and Kozicki (1993), we will focus our attention on features which are present in the analysed series but there exists at least one linear combination of these series which does not possess these features. One of the most famous examples of this idea is cointegration. When a group of the series shares common stochastic trends, we say that they are cointegrated, so there exists a linear combination of them which lowers the order of integration of the analysed series. Another example of the cofeature is a serial correlation common feature. In such case there exists a linear combination of the series which is an innovation with respect to past information. In 1993 Vahid and Engle considered cointegration for the levels of I(1) series and a serial correlation common feature for their first differences, together in one model. In the VEC model the serial correlation common feature leads to an additional reduced rank restriction imposed on the parameters.
Let us consider the n-dimensional conintegrated process x_t and write it in the VEC form:

$$\Delta x_t = \alpha\beta' x_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-i} + \Phi D_t + \varepsilon_t = \alpha\beta' x_{t-1} + \Gamma' z_t + \Phi D_t + \varepsilon_t, \tag{1}$$

where $z_t' = (\Delta x_{t-1}', \Delta x_{t-2}', \ldots, \Delta x_{t-k+1}')$, $\Gamma = (\Gamma_1, \ldots, \Gamma_{k-1})'$, $\varepsilon_t \sim iiN^n(0, \Sigma), t = 1, 2, \ldots, T$.
In the case of the common serial correlation among the first differences of the series all Γ's and α must have less than full rank and their left null spaces must overlap (Vahid, Engle 1993), which leads to the following model:

$$\Delta x_t = \gamma^* \delta_0^{*'} \beta' x_{t-1} + \sum_{i=1}^{k-1} \gamma^* \delta_i^{*'} \Delta x_{t-i} + \Phi D_t + \varepsilon_t = \gamma^* \delta^{*'} z_t^* + \Phi D_t + \varepsilon_t, \tag{2}$$

where $\delta^{*'} = (\delta_0^{*'}, \delta_1^{*'}, \ldots, \delta_{k-1}^{*'})$, $z_t^* = (x_{t-1}'\beta, \Delta x_{t-1}', \ldots, \Delta x_{t-k+1}')'$. The matrices $\gamma^*_{n\times(n-s)}$ and $\delta^*_{(n-s)\times(n(k-1)+r)}$ have full rank.
There exist s linear combinations of the process $\Delta x_t - \Phi D_t$ which are innovations: $\gamma_\perp^{*'}(\Delta x_t - \Phi D_t) = \gamma_\perp^{*'} \varepsilon_t$.
Any cointegrated process can be written as the sum of a multivariate random walk (τ_t), a stationary process (κ_t) and initial values (δ_t), which is known as a multivariate Beveridge-Nelson decomposition (see e.g. Johansen 1996, Lütkepohl 2007):

$$x_t = \delta_t + \tau_t + \kappa_t = \delta_t + C(1) \sum_{i=0}^{t-1} \varepsilon_t + C^*(L)\varepsilon_t, \tag{3}$$

where $C(1) = \beta_\perp(\alpha'_\perp(I_n - \sum_{i=1}^{k-1}\Gamma_i)\beta_\perp)^{-1}\alpha'_\perp$, $C^*(L)\varepsilon_t = \sum_{j=0}^{\infty} C_j^*\varepsilon_{t-j}$.
Vahid and Engle (1993) showed that $\gamma_\perp^{*'}\kappa_t = 0$, so the $\gamma_\perp^*$ matrix cancels both past information of the series Δx_t and the stationary part of x_t, which is called the cycle (κ_t), so the process x_t has s common cycles. For this reason, this type of comovement is an example of the Common Cyclical Features idea.
The above presented type of short-run comovements is very strong and the number of common serial correlation features cannot exceed the number of common trends $(n-r)$. In 2006 Hecq, Palm and Urbain introduced a model with the so called weak form reduced rank structures which do not place limitations on the number of common features. In this case there exists a linear combination of the first differences adjusted for long-run effects, which is an innovation. This restriction implies the reduced rank structures only on the matrices of short part of the model, i.e. on Γ's:

$$\Delta x_t = \alpha\beta' x_{t-1} + \gamma\delta'_1\Delta x_{t-1} + \cdots + \gamma\delta'_{k-1}\Delta x_{t-k+1} + \Phi D_t + \varepsilon_t = \alpha\beta' x_{t-1} + \gamma\delta' z_t + \Phi D_t + \varepsilon_t, \tag{4}$$

where $\delta' = (\delta'_1, \delta'_2, \ldots, \delta'_{k-1})$. The $\gamma_{n\times(n-s)}$ and $\delta_{n(k-1)\times(n-s)}$ matrices are of full column rank.
There exist s linear combinations of the process $\Delta x_t - \alpha\beta' x_{t-1} - \Phi D_t$, which are innovations. In the case of the weak common cyclical features the short- and long-run dynamics are unrelated contrary to the strong case where they are similar.
Hecq, Palm and Urbain (2006) showed that such definition is not invariant to alternative VEC models reparametrisations such as those where $\beta' x_{t-p}$ appears instead of $\beta' x_{t-1}$.
Cubadda (2007) showed that in the case of weak form of the serial correlation common feature there exists a first order polynomial matrix $\gamma_\perp(L) \equiv \gamma_\perp - (\beta\alpha' + I_n)\gamma_\perp L$ such that $\gamma_\perp(L)' x_t = \gamma'_\perp \Phi D_t + \gamma'_\perp \varepsilon_t$ and $\gamma_\perp(L)'\kappa_t = \gamma'_\perp(I_n - C(1))\varepsilon_t$, so it cancels the dependence from the past of both the cycles (κ_t) and the series x_t adjusted for deterministic terms (see also Centoni, Cubadda 2011).
Ericsson (1993) pointed out that it will be useful to consider also non-contemporaneous relations among the analysed times series, which are excluded under the assumptions of the serial correlation common features. In 1997 Vahid and Engle proposed the model and the test for the non-synchronised comovement of the processes. Later, Cubadda and Hecq (2001) introduced the concept of the polynomial serial correlation common features which allows us to describe non-contemporaneous short-run comovements among the first differences of the integrated processes.
The series Δx_t have s polynomial serial correlation common features of order 1, iff there exists an $n \times s$ full column matrix ψ_P^* such that $\psi_P^{*'}\Gamma_1 \neq 0$, and the VEC model can be rewritten in the following form:

$$\Delta x_t = \Gamma_1\Delta x_{t-1} + \gamma_P^*\delta_P^{*'}(x'_{t-1}\beta, \Delta x'_{t-2}, \ldots, \Delta x'_{t-k+1})' + \Phi D_t + \varepsilon_t, \tag{5}$$

where $\gamma_P^* = \psi_{P\perp}^*$.
In this case there exists a polynomial matrix $\psi^*(L) = \psi_P^* - \Gamma'_1\psi_P^* L$ which cancels the

dependence form the past of the process Δx_t, i.e. $\psi^*(L)'\Delta x_t = \psi_P^{*'}\Phi D_t + \psi_P^{*'}\varepsilon_t$. The same polynomial matrix transforms the cyclical part of the series x_t into an innovation process: $\psi^*(L)'\kappa_t = -\psi_P^{*'}\Gamma_1 C(1)\varepsilon_t$ (see Cubadda, Hecq 2001 and Centoni, Cudadda 2011).

It is of course possible to merge the weak form of serial correlation common feature and the polynomial serial correlation common feature. This way we obtain the weak form of the polynomial serial correlation common feature (Cubadda 2007).

The series Δx_t have s weak form of polynomial serial correlation common features of order 1 (WFP(1)), iff there exists an $n \times s$ full column matrix ψ_P such that $\psi_P^{'}\alpha \neq 0$, $\psi_P^{'}\Gamma_1 \neq 0$, and the VEC model can be rewritten in the following form:

$$\Delta x_t = \alpha\beta' x_{t-1} + \Gamma_1 \Delta x_{t-1} + \gamma_P \delta_P^{'}(\Delta x'_{t-2}, \ldots, \Delta x'_{t-k+1})' + \Phi D_t + \varepsilon_t, \qquad (6)$$

where $\gamma_P = \psi_{P\perp}$.

In this case there exists a polynomial matrix $\psi_P(L) = \psi_P - (\beta\alpha' + I_n + \Gamma_1')\psi_P L + \Gamma_1'\psi_P L^2$ such that $\psi_P(L)'x_t = \psi_P'\Phi D_t + \psi_P'\varepsilon_t$, so it cancels the dependence of x_t from the past. This polynomial matrix also transforms the cycles of the series x_t into a VMA(1) process, so into a process with shorter memory: $\psi_P(L)'\kappa_t = \psi_P'[I_n - C(1)]\varepsilon_t - \psi_P'\Gamma_1 C(1)\varepsilon_{t-1}$ (see Cubadda 2007 and Centoni, Cudadda 2011).

The definitions of polynomial serial correlation common features may be extended for higher orders (see e.g. Cubadda, Hecq 2001).

In the present paper we are interested in the Bayesian analysis of the last form of the above described common features. In the next Section the Bayesian VEC-WFP model is introduced, Section 3 presents the analysis of price-wage nexus in the Polish economy based on that model and the final Section concludes.

2 The Bayesian VEC model with weak form of polynomial serial correlation common features

In this section we will focus our attention on the Bayesian analysis of the weak form of polynomial serial correlation common features of order p which will be conducted via the model of the following form:

$$\Delta x_t = \alpha\beta_+' x_{t-1}^+ + \Gamma' w_t + \gamma_P \delta_P^{'} z_t + \Phi D_t + \varepsilon_t, \;\; \varepsilon_t \sim iiN^n(0, \Sigma), \;\; t = 1, 2, \ldots, T \quad (7)$$

where $w_t = (\Delta x'_{t-1}, \ldots, \Delta x'_{t-p})'$, $z_t = (\Delta x'_{t-p+1}, \ldots, \Delta x'_{t-k+1})'$, $\beta_+ = (\beta' \phi')'$, $x_{t-1}^+ = (x'_{t-1} \; d_t)'$. The term d_t incorporates deterministic components into cointegrating relations. To simplify the notation let us write the basic model (7) in a matrix form:

$$Z_0 = Z_1\beta_+\alpha' + W\Gamma + Z_2\delta_P\gamma_P' + Z_3\Gamma_s + E = Z_1\Pi' + W\Gamma + Z_2\Gamma_P + Z_3\Gamma_s + E, \quad (8)$$

where $Z_0 = (\Delta x_1, \Delta x_2, \ldots, \Delta x_t)'$, $Z_1 = (x_0^+, x_1^+, \ldots, x_{T-1}^+)'$, $W = (w_1, w_2, \ldots, w_T)'$, $Z_2 = (z_1, z_2, \ldots, z_T)'$, $Z_3 = (D_1, D_2, \ldots, D_T)'$, $\Gamma_s = \Phi'$ and $E = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T)'$.

In this model we have two reduced rank matrices (Π and Γ_P) which are decomposed as the products of full rank matrices, i.e. $\Pi = \alpha\beta'_+$ and $\Gamma_P = \delta_P\gamma'_P$. It is commonly known that such decomposition is not invariant, i.e. for any full rank matrices of adequate dimensions, C_Π and C_Γ, the following equalities are obtained: $\alpha\beta' = \alpha C_\Pi C_\Pi^{-1}\beta'$, $\gamma_P\delta'_P = \gamma_P C_{\Gamma_P} C_{\Gamma_P}^{-1}\delta'_P$. For this reason we should estimate spaces spanned by β and δ matrices (which are elements of the Grassmann manifolds) rather than these matrices. We have decided to use the scheme of estimation proposed by Koop, León-González and Strachan (2010) for VEC models, which takes into account the curved geometry of the parameters and at the same time allows for the use of the parameter-augmented Gibbs sampling scheme to sample from the posterior distribution. In the context of the VEC models with an additional reduced rank restriction this scheme was employed by Wróblewska (2011).
For Π and Γ_P two parametrisations will be used:

$$\alpha\beta' = (\alpha M_\Pi)(\beta M_\Pi^{-1})' \equiv AB', \tag{9}$$

where M_Π is an $r \times r$ symmetric positive definite matrix, and

$$\gamma\delta' = (\gamma M_{\Gamma_P})(\delta M_{\Gamma_P}^{-1})' \equiv G_P D'_P, \tag{10}$$

where M_{Γ_P} is a $q \times q$ symmetric positive definite matrix.
We assume that A, B, G_P and D_P are unrestricted matrices ($A \in \mathbb{R}^{nr}$, $B \in \mathbb{R}^{mr}$, $G_P \in \mathbb{R}^{nq}$, $D_P \in \mathbb{R}^{lq}$,), whilst β and δ_P have orthonormal columns, i.e. they are elements of the Stiefel manifolds: $\beta \in V_{r,m}$, $\delta_P \in V_{q,l}$. Through these matrices we want to get information about the spaces. The relationship between the Stiefel and Grassmann manifolds is many-to-one, i.e. in each point of the Grassmann manifold is contained a set of elements of the Stiefel manifold. To weaken this drawback we additionally assume that elements of the first row of β and δ_P are positive: $\beta \in \tilde{V}_{r,m}$, $\delta_P \in \tilde{V}_{q,l}$, where $\tilde{V}_{r,m}$ denotes the 2^{-r}th part of $V_{r,m}$ and $\tilde{V}_{q,l}$ - the 2^{-q}th part of $V_{q,l}$ (Chikuse 2002). The invariant measures over $\tilde{V}_{r,m}$ and $\tilde{V}_{q,l}$ differ form the invariant measures over $V_{r,m}$ and $V_{q,l}$ by multiplicative constants 2^r and 2^q respectively.
Imposing matrix Normal distributions on B and D_P ($mN(0, I_r, P_B)$, $mN(0, I_q, P_D)$) leads to Matrix Angular Central Gaussian distributions for the orientations of B and D_P: $B(B'B)^{-\frac{1}{2}} \sim MACG(P_B)$ and $D_P(D'_P D_P)^{-\frac{1}{2}} \sim MACG(P_D)$. Through the matrices P_B and P_D a researcher may incorporate prior information about the estimated spaces. Prior distributions for the remaining parameters are standard: $A|\Sigma,\nu_A,r \sim mN(0,\nu_A I_r,\Sigma)$, $G_P|\Sigma,\nu_{G_P},q \sim mN(0,\nu_{G_P} I_q,\Sigma)$, $\nu_A \sim iG(s_{\nu_A},n_{\nu_A})$, $\nu_{G_P} \sim iG(s_{\nu_{G_P}},n_{\nu_{G_P}})$, $\Sigma \sim iW(S, q_\Sigma)$, $\Gamma|\Sigma, h \sim mN(0, \Sigma, hI)$, $h \sim iG(s_h, n_h)$, $\Gamma_s|\Sigma, h_s \sim mN(0, \Sigma, h_s I)$, $h_s \sim iG(s_{h_s}, n_{h_s})$.
The joint prior distribution is truncated by the stability condition imposed on the parameters of the process:
$p(A,B,G_P,D_P,\Sigma,\Gamma,\Gamma_s,\nu_A,\nu_{G_P},h,h_s) \propto f(p(A,B,G_P,D_P,\Sigma,\Gamma,\Gamma_s,\nu_A,\nu_{G_P},h,h_s) I_{[0,1]}(|\lambda|_{max})$, where λ denotes the eigenvalue of the companion matrix.

After incorporating into the model the information contained in the data one gets the following posterior distributions (for the parametrisation with B and D_P):

- $iW(S + \frac{1}{h}\Gamma'\Gamma + \frac{1}{h_s}\Gamma_s'\Gamma_s + \frac{1}{\nu_A}AA' + \frac{1}{\nu_{G_P}}G_P G_P' + E'E, q_\Sigma + pn + l_s + r + q + T)$ for Σ, where $E = Z_0 - Z_1 BA' - Z_2 D_P G_P' - W\Gamma - Z_3\Gamma_s$,
- $mN(\mu_{\Gamma_s}, \Sigma, \Omega_{\Gamma_s})$, for Γ_s, where $\mu_{\Gamma_s} = (\frac{1}{h_s}I_{l_s} + Z_3'Z_3)^{-1}Z_3'(Z_0 - Z_1BA' - Z_2D_PG_P' - W\Gamma)$, $\Omega_{\Gamma_s} = (\frac{1}{h_s}I_{l_s} + Z_3'Z_3)^{-1}$
- $mN(\mu_\Gamma, \Sigma, \Omega_\Gamma)$, for Γ, where $\mu_\Gamma = (\frac{1}{h}I_{np} + W'W)^{-1}W'(Z_0 - Z_1BA' - Z_2D_PG_P' - Z_3\Gamma_s)$, $\Omega_\Gamma = (\frac{1}{h}I_{np} + W'W)^{-1}$
- $mN(\mu_A, (\frac{1}{\nu_A}I_r + B'Z_1'Z_1B)^{-1}, \Sigma)$, for A, where $\mu_A = (Z_0 - Z_2DG' - W\Gamma - Z_3\Gamma_s)'Z_1B(\frac{1}{\nu_G}I_r + B'Z_1'Z_1B)^{-1}$,
- the Normal with variance $\Omega_{vB} = ([(A'\Sigma^{-1}A) \otimes (Z_1'Z_1)] + [mI_r \otimes P_B])$ and mean $\mu_{vB} = \Omega_{vB}vec(Z_1'(Z_0 - Z_2DG' - W\Gamma - Z_3\Gamma_s)\Sigma^{-1}A)$ for $vec(B)$,
- $mN(\mu_{G_P}, (\frac{1}{\nu_{G_P}}I_q + D_P'Z_2'Z_2D_P)^{-1}, \Sigma)$ for G_P, where $\mu_{G_P} = (Z_0 - Z_1BA' - W\Gamma - Z_3\Gamma_s)'Z_2D_P(\frac{1}{\nu_{G_P}}I_q + D_P'Z_2'Z_2D_P)^{-1}$,
- the Normal with variance $\Omega_{vD} = ([(G_P'\Sigma^{-1}G_P) \otimes (Z_2'Z_2)] + [lI_q \otimes P_D])$ and mean $\mu_{vD} = \Omega_{vD}vec(Z_2'(Z_0 - Z_1BA' - W\Gamma - Z_3\Gamma_s)\Sigma^{-1}G_P)$ for $vec(D_P)$,
- inverted gamma distributions: $iG(s_{\nu_A} + \frac{1}{2}tr(\Sigma^{-1}AA'), n_{\nu_A} + \frac{nr}{2})$ for ν_A, $iG(s_{\nu_{G_P}} + \frac{1}{2}tr(\Sigma^{-1}GG'), n_{\nu_{G_P}} + \frac{nq}{2})$ for ν_{G_P}, $iG(s_h + \frac{1}{2}tr(\Sigma^{-1}\Gamma\Gamma'), n_h + \frac{np}{2})$ for h and $iG(s_{h_s} + \frac{1}{2}tr(\Sigma^{-1}\Gamma_s\Gamma_s'), n_{h_s} + \frac{nl_s}{2})$ for h_s.

Draws from the posterior distribution of β, α are obtained as $\beta = B(B'B)^{-\frac{1}{2}}$, $\alpha = A(B'B)^{\frac{1}{2}}$, δ_P and γ_P are obtained as $\delta_P = D_P(D_P'D_P)^{-\frac{1}{2}}$, $\gamma_P = G_P(D_P'D_P)^{\frac{1}{2}}$.Having the sample from the posterior distribution the mean of β and δ_P can be computed with the method proposed by Villani (2006), i.e. by constructing the loss function, which takes the curved geometry of the Grassmann manifold into account, e.g. with the projective Frobenius distance between spaces.

3 An empirical illustration: the price-wage nexus in the Polish economy

The presented methods will be illustrated with the analysis of the price - wage spiral in the Polish economy. The seasonally unadjusted quarterly data represent five variables: average wages (current prices, W_t), price index of consumer goods (P_t), labour productivity (constant prices, Z_t), price index of imported goods (M_t) and the unemployment rate (U_t). The analysed data cover the sixteen-year period ranging from 1995Q1 to 2010Q4. The data are plotted in Figure 1. The visual inspection of the analysed variables suggests that they may be realisations of the integrated processes, but they appear to move together in the long-run, so we can expect cointegration. The first differences of the series also seem to show a similar short-run behaviour, so it is reasonable to verify the hypothesis of the additional reduced rank restriction imposed on the short-run parameters of the VEC model. As shown by Fisher (1997) and Taylor (1980), the comovement between wages and prices may be unsynchronised (see also Vahid, Engle 1997), which is caused by wage contracts lasting more than one period. Our task is to verify this hypothesis for the Polish economy.

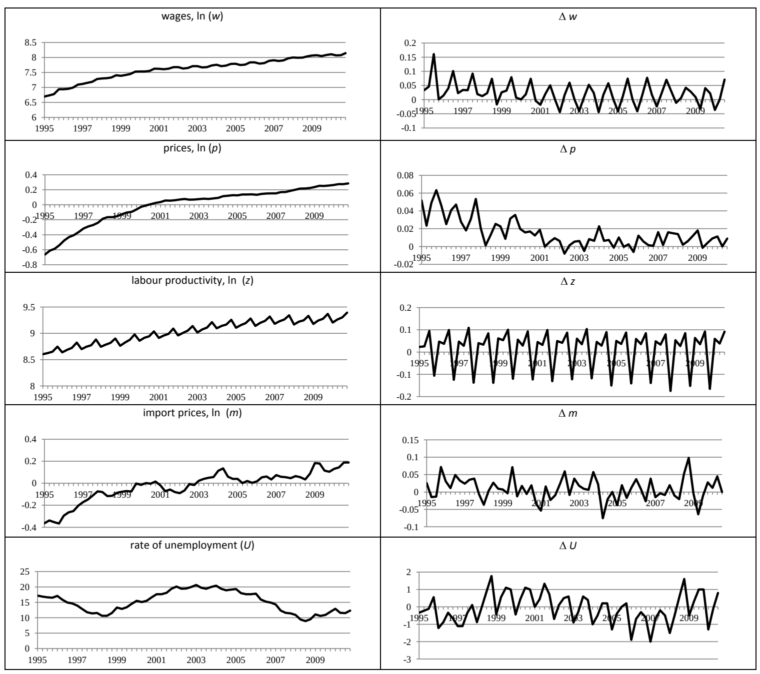

Figure 1: The analysed data

We will consider a set of models which differ in the number of lags $k \in \{3,4,5\}$, deterministic terms $d \in \{1,2\}$, where $d = 1$ stands for an unrestricted constant, $d = 2$ - a constant restricted to cointegrating relations (see e.g. Juselius 2007 for further details), the number of cointegrating relations $r \in \{1,2,3,4\}$, the number of (polynomial) weak common cyclical features $s \in \{0,1,2,3,4\}$ (i.e the rank of Γ_P: $n - s = q \in \{5,4,3,2,1\}$, for $s = 0$ we have a VEC model) and the number of quarters

previous to short-run comovements $p \in \{0,1,2\}$ (for $p = 0$ the weak common cyclical features are synchronised).
The seasonality of the analysed series will be modelled in the deterministic manner, i.e. via zero-mean seasonal dummies.
Altogether we will compare 280 different specifications of VEC-(P)WF model: 24 - VEC, 96 VEC-WF, 96 VEC-PWF(1) and 64 VEC-PWF(2) models. As we want to treat them as equally possible we impose on each of them the same prior probability: $p(M_{(k,d,q,r,p)}) = \frac{1}{280} \approx 0.0036$.
We impose the following priors on the model parameters:

- $\Sigma \sim iW(S, 10+n+1),\ S = 10\begin{pmatrix} 0.05 & 0 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 & 0 \\ 0 & 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0.05 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$,
- $B|r, m \sim mN(0, m^{-1}I_r, I_m)$, which leads to $\beta|r \sim MACG(I_m)$,
- $A|\nu_A, r, \Sigma \sim mN(0, \nu_A I_r, \Sigma)$,
- $D_P|q, l \sim mN(0, l^{-1}I_q, I_l)$, which leads to $\delta_P|q \sim MACG(I_l)$,
- $G_P|\nu_{G_P}, q, \Sigma \sim mN(0, \nu_{G_P} I_q, \Sigma)$,
- $\Gamma|\Sigma, h \sim mN(0, \Sigma, hI)$,
- $\Gamma_s|\Sigma, h_s \sim mN(0, \Sigma, h_s I)$,
- $\nu_A \sim iG(2,3)$ $(E(\nu_A) = 1,\ Var(\nu_A) = 1)$,
- $\nu_{G_P} \sim iG(2,3)$ $(E(\nu_{G_P}) = 1,\ Var(\nu_{G_P}) = 1)$,
- $h \sim iG(20,3)$ $(E(h) = 10,\ Var(h) = 100)$,
- $h_s \sim iG(20,3)$ $(E(h_s) = 10,\ Var(h_s) = 100)$,

The joint prior resulting from this specification has been truncated by the stability condition imposed on the parameters of the cointegrated process.
The most probable models (i.e. with posterior probability not lower than 0.01) are presented in Table 1. The sum of posterior probabilities of the listed models equals 0.742. Table 2 presents marginal probabilities of model features.
Table 1 shows us great posterior model uncertainty, but from Table 2 we can draw the conclusion that, in the analysis of price-wage spiral in the Polish economy, we should take into consideration the possibility of both long- and short-run comovements of the observed data. Contrary to our presumption, models with immediate short-run comovements achieved more posterior probabilities than that with delayed short-run common behaviour.

Table 1: The most probable models, $p(M_{(k,d,q,r,p)}|x) > p(M_{(k,d,q,r,p)})$

k	d	q	r	p	$log_{10}(\hat{p}(x|M_{(k,d,q,r,p)}))$	$p(M_{(k,d,q,r,p)}|x)$
4	2	1	3	0	106.587	0.041
4	2	1	2	0	106.531	0.036
4	2	1	4	0	106.525	0.035
3	2	3	1	0	106.494	0.033
4	2	3	3	0	106.491	0.033
3	2	2	4	0	106.447	0.029
3	2	3	2	0	106.434	0.029
3	2	4	1	0	106.428	0.028
3	2	2	2	0	106.419	0.028
3	2	3	3	0	106.416	0.027
3	2	2	3	0	106.316	0.022
3	2	4	2	0	106.307	0.021
4	2	4	3	0	106.292	0.021
3	2	1	4	0	106.292	0.021
3	2	1	1	1	106.275	0.020
3	2	1	3	1	106.264	0.019
3	2	4	4	0	106.253	0.019
3	2	1	1	0	106.242	0.018
3	2	1	2	1	106.240	0.018
3	2	1	3	0	106.240	0.018
3	1	1	1	0	106.226	0.018
4	2	1	2	1	106.209	0.017
4	2	2	3	0	106.194	0.016
4	2	1	1	1	106.187	0.016
4	1	2	3	0	106.170	0.016
3	2	4	3	0	106.131	0.014
3	2	2	3	1	106.122	0.014
4	2	2	2	1	106.096	0.013
4	2	1	3	1	106.088	0.013
4	2	2	3	1	106.078	0.013
4	2	2	1	0	106.065	0.012
4	2	4	1	0	106.055	0.012
3	2	2	2	1	106.036	0.011
4	1	1	2	0	106.008	0.011
5	2	1	2	1	106.005	0.011
4	2	2	1	1	105.997	0.010
4	2	4	2	0	105.990	0.010

Table 2: Marginal posterior probabilities of model features

k	3	4	5		
$p(k\|x)$	0.493	0.394	0.113		
d	1	2			
$p(d\|x)$	0.116	0.884			
r	1	2	3	4	
$p(r\|x)$	0.260	0.260	0.317	0.164	
q	1	2	3	4	5
$p(q\|x)$	0.397	0.256	0.178	0.157	0.013
p	0	1	2		
$p(p\|x)$	0.690	0.308	0.002		

Figure 2 compares forecasts obtained with the Bayesian model averaging technique in the analysed model classes, i.e. VEC, VEC-WF and VEC-WFP(1). Figures 3 and 4 present chosen impulse response functions.

The visual analysis of the presented Figures 2 and 3 point to the conclusion that as the imposed short-run restrictions lead to more parsimonious models, they also improve the precision of further analysis. As should be expected, the additional restrictions change shapes of the impulse response functions in the short-run periods (see Figure 4).

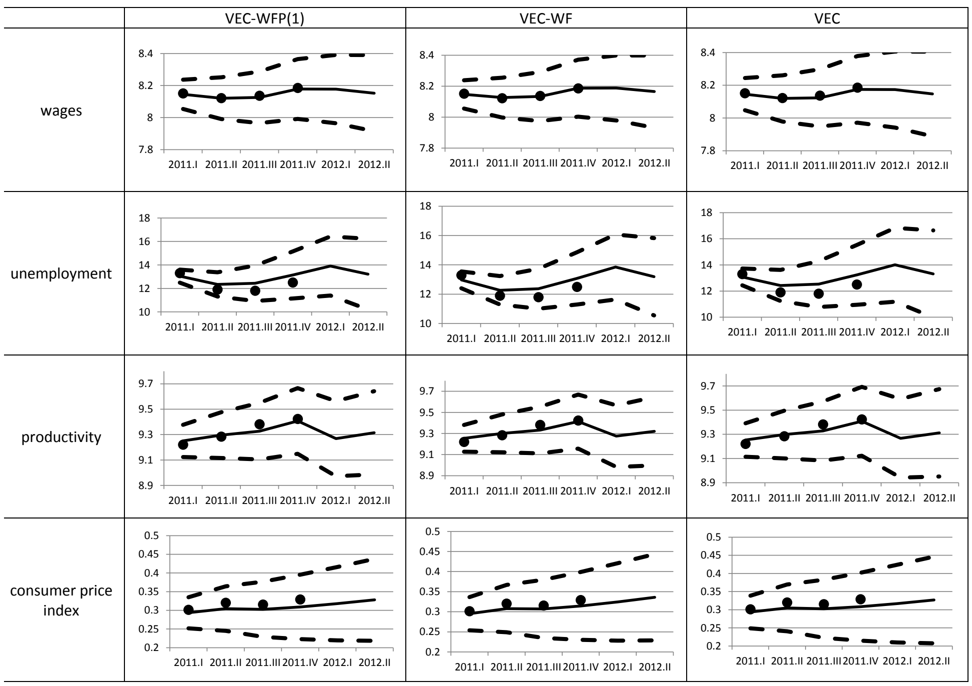

Figure 2: Predictive means (solid lines) and standard deviations (dashed lines) in model groups VEC-WF and VEC-WFP(1) and VEC compared to true values of forecasted variables (dots).

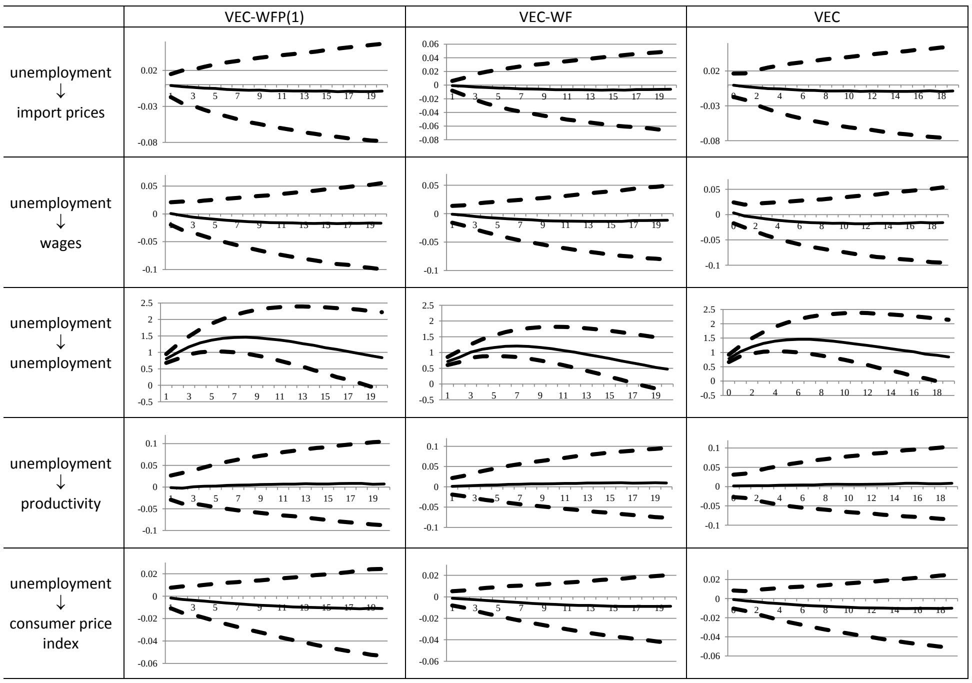

Figure 3: Posterior results of impulse responses of analysed variables to shock in unemployment. The solid line represents the posterior median and the dotted lines are the 10^{th} and 90^{th} percentiles.

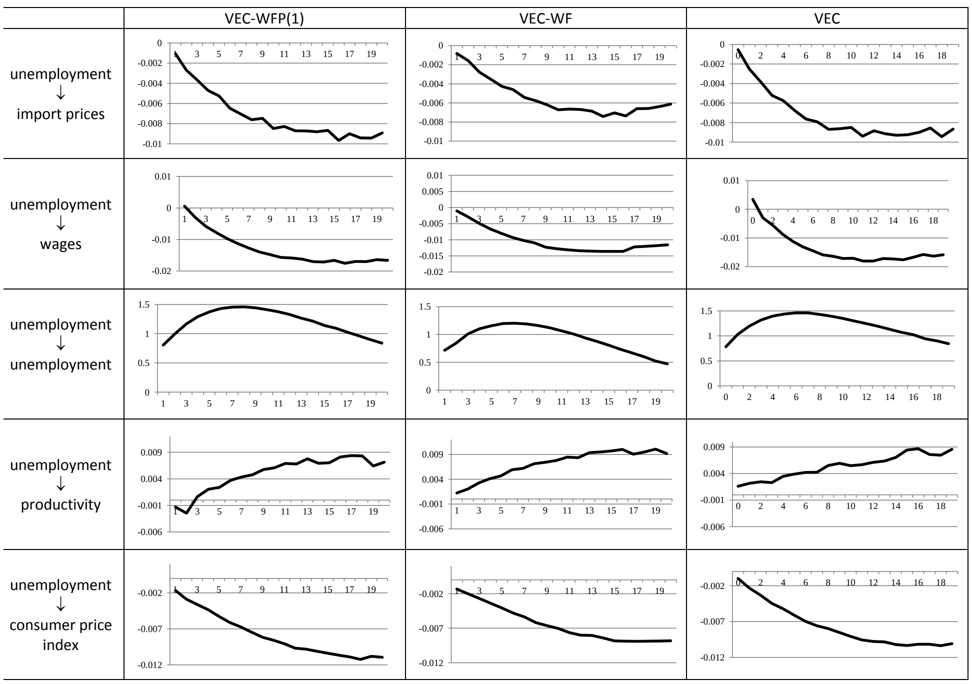

Figure 4: Posterior medians of impulse responses of analysed variables to shock in unemployment.

4 Concluding remarks

In this paper we proposed a Bayesian treatment (i.e. estimation and comparison) of VEC models with the additional weak form (polynomial) reduced rank restriction imposed on the short-run parameters of such models. In the empirical example we used the proposed method to analyse the price - wage spiral in the Polish economy. The Bayesian comparison of the models confirmed the hypothesis of the presence of long-run and short-run relations among the analysed variables, but the non-synchronised short-run comovement is only weakly supported.
Additionally, we showed the consequences of such restrictions for forecasting and for impulse response analysis of the VEC-WF(P) system.

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