Posterior-Predictive Evidence on US Inflation using a New Keynesian Phillips Curve with Weak Identification, Regime Shifts and Technological Change

Nalan Baştürk[1], Pınar Ceyhan[1,2], Cem Çakmaklı[*,3], and Herman K. van Dijk[1,2,4]

[1] *Econometric Institute, Erasmus University Rotterdam*
[2] *Tinbergen Institute*
[3] *Department of Quantitative Economics, University of Amsterdam*
[4] *VU University Amsterdam*

July 2012
(preliminary paper; not to be quoted or distributed)

Abstract

In this paper the changing time series properties of US inflation and marginal costs are incorporated in a NKPC using structural time series modeling. The misspecification effects of mechanical removal of low frequency movements of these series on the forecasting performance of the model are analyzed using a Bayesian simulation based approach. Changing patterns in high frequency (volatility) are also incorporated in the enlarged NKPC model. Special attention is paid to the issue of weak identification in the model with filtered and observed time series. Posterior and predictive results show a systematic bias in parameter estimates using the filtered series; reduction of the weak identification issue in the enlarged NKPC model and more precise inference of the structural NKPC parameters.

Keywords: New Keynesian Philips curve, unobserved components, detrending, level shifts
JEL Classification: C11, C31, C36, E37

[*]Correspondence to: Cem Çakmaklı, Valckenierstraat 65-67, 1018 XE Amsterdam, The Netherlands, e–mail: C.Cakmakli@uva.nl.

1 Introduction

The relation between inflation and output fluctuations has been attracted great interest in macroeconomic analysis since this interaction of these variables provides a useful tool for policy analysis. The New Keynesian Philips Curve (NKPC), which relates the dynamics of short run inflation to economic activity, is the building block of many macro models with micro foundations, such as the Dynamic Stochastic General Equilibrium (DSGE) models. The conventional way to estimate NKPC models is to filter, i.e. demean and/or detrend the series a priori (see Galí and Gertler (1999); Mavroeidis (2004); DeJong and Dave (2011) among others). However, the complex time series structure of the macroeconomic series obscure econometric inference based on these demeaned and detrended series. The existence of complex low frequency movements, such as potential structural breaks and level shifts in the observed series, require more sophisticated models, which can handle these time variation together with the standard NKPC parameters.

The purpose of this paper is to model the low and high frequency movements in the inflation and marginal cost series jointly by extending the standard NKPC model by modeling the observed time series instead of the filtered series. Posterior and predictive results for the proposed model are obtained using a simulation based Bayesian approach. We compare the results of this extended model with those of the standard NKPC model with demeaned and/or detrended data in order to analyze the misspecification effects on inference and on forecasting performance. We focus particularly on the issue of weak identification in the NKPC model with filtered and observed time series. Apart from the low frequency movements in the data we also include changing patterns in high frequency moments in an enlarged NKPC model, by incorporating a stochastic volatility structure for inflation.

Conventional econometric analysis of the NKPC model is based on demeaned and detrended data. The reason of this a priori data transformation is that the

NKPC applies to the series in deviation from the steady state. On the one hand, there is no consensus on the appropriate method of detrending these series, see Gorodnichenko and Ng (2010) for a comprehensive list of such methods used in the literature. The econometric analysis and the policy implications may be sensitive to the particular detrending method used. On the other hand, this mechanical removal of the low frequency movements in the data may lead to misspecification in the models, as suggested in Canova (2012) for DSGE models. Specifically, more complex high frequency moments in the data, such as long-run trends or regime switching behavior, are not accounted for in such a priori analysis. The existence of such complex low frequency moments, in particular in the inflation series, are documented extensively in the literature (McConnell and Perez-Quiros, 2000; Stock and Watson, 2008; Zhang et al., 2008; Bianchi, 2010). A misspecification in the model due to such non-standard long-run behavior of the data may explain part of the weak identification or rank reduction issues in NKPC reported e.g. in Mavroeidis (2004). The resulting inflation forecasts may as well be affected by such misspecification.

Existing evidence suggests such complex long-run behavior in many macroeconomic series too. For instance two distinct periods with differing patterns can be observed for the raw inflation series. The period between the beginning of 1970s and beginning of 1980s is often labeled as a high inflationary period compared to the latter periods. The decline in the level and volatility after this period is linked to credible monetary policy that stabilized inflationary expectations at a low level via commitment to a nominal anchor since the early eighties, see McConnell and Perez-Quiros (2000); Stock and Watson (2002); Ahmed et al. (2004); Stock and Watson (2007); Cecchetti et al. (2007).

Following these concerns, we extend the NKPC model to explicitly incorporate trends and low frequency movements observed in the series. As a result, we estimate these long run behavior of the series along with the NKPC model parameters using structural time series techniques. For the inflation series we impose a specification which can handle rare and large shifts in the level of inflation. For the series representing economic activity, we use the labour share series as a proxy of the real marginal cost in the economy. This series is directly linked to the theory underlying NKPC and it is found to be useful indicator of economic activity, see Galí and Gertler (1999); Clarida et al. (2000). For the trend of this series, we propose a general specification which also includes the mechanical detrending techniques as special cases.

We apply the proposed model to quarterly U.S. data over the period between between the first quarter of 1960 and the last quarter of 2011. We show that due to the misspecification issue in the standard NKPC model, there is a systematic bias in parameter estimates using the filtered series; a reduction of the weak identification issue, see Mavroeidis (2004, 2005); Kleibergen and Mavroeidis (2009, 2011), in the enlarged NKPC model and more precise inference of the structural NKPC parameters. The proposed model captures time variation in the low frequency moments of both inflation and marginal cost data. For the inflation series, the model identifies two distinct periods with differing inflation levels. The relatively high inflationary period spans the period between the beginning of 1970s and beginning of 1980s. This period is replaced rapidly by a relatively low inflation period, where annual inflation is anchored at a level around 2%, accompanying the changing monetary policy in the U.S.. This changing behavior of the inflation levels cannot be accurately captured by the conventional NKPC models using a priori filtered data. In terms of the marginal cost series, the trend specification accommodates the smoothly changing trend observed in the series, specifically after 2000.

The remainder of this paper is as follows: Section 2 illustrates the effects of misspecified low frequency moments on inference and prediction using a canonical AR model with long-run trends. Section 3 presents the standard NKPC model and identification issues in the inference of the model parameters. Section 4 presents the extended NKPC models. Section 6 provides the application of the proposed models and the standard NKPC model on US income and marginal cost data. Section 7 concludes.

2 Misspecified structural breaks

The purpose of this section is to illustrate potential issues from misspecified structural breaks in empirical analysis, using simulated data and a canonical model with structural breaks. Specifically, we show that a misspecification resulting from ignoring structural breaks in the data lead to an overestimation of the data persistence. We further show that this overestimation in persistence deteriorates the forecast performance.

For the illustrations, we consider a canonical AR(2) model with a break structure in the long-run mean:

$$\begin{aligned} z_t &= c_t + \upsilon_t, \\ c_t &= c_{t-1} + \kappa_t \eta_{1,t}, \\ \upsilon_t &= \phi_1 \upsilon_{t-1} + \phi_2 \upsilon_{t-2} + \eta_{2,t}, \end{aligned} \tag{1}$$

for $t = 1, \ldots, T$, where z_t is the data modeled by two unobserved components: a time-varying mean c_t, and a transitionary component υ_t.

In equation (1), ϕ_1 and ϕ_2 are model parameters, κ_t is a binary variable which takes the value 1 if the mean changes at time t, and 0 otherwise, where κ_t has a binomial distribution with probability $p(\kappa = 1) = p(\kappa) \in [0, 1]$ and $\eta_{1,t}$ and $\eta_{2,t}$ are the residuals with $(\eta_{1,t}, \eta_{2,t})' \sim NID\left(0, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}\right)$. Note that the mean of the process is non-stationary if $\exists t$ such that $\kappa_t = 1$. Furthermore, if $\kappa_t = 1$ for $t = 1, \ldots, T$, the model in (1) is an AR(2) model with a random walk structure in the mean.

Suppose that the model in (1) is modeled without taking the time-dependency in the mean into account. The model can then be estimated focusing on the demeaned data $\tilde{z}_t = z_t - \sum_{t=1}^{T} z_t/T$:

$$\tilde{z}_t = \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \epsilon_t, \tag{2}$$

where $\epsilon_t \sim NID(0, \sigma_\epsilon^2)$ and standard stationarity conditions $\phi_1 + |\phi_2| < 1$ and $|\phi_1| < 1$ apply.

We simulate data with different structural break probabilities $p(\kappa) = \{0, 0.05, 0.1\}$ and different persistence parameters ϕ_1, ϕ_2, with $T = 200$ observations to illustrate overestimation in persistence under the misspecified model. The rest of the model parameters are set as $\sigma_1^2 = 4, \sigma_2^2 = 1, c_0 = 1, \nu_0 = \nu_{-1} = 2$. Note that for the simulated data with $p(\kappa) = 0$, the model in (2) is correctly specified, and the posterior results should be close to the true values.

For each simulated dataset we estimate the model in (2) under uninformative priors:

$$p(\phi_1, \phi_2, \sigma_\epsilon^2) \propto \begin{cases} 1 & \text{if } \sigma_\epsilon^2 > 0,\ \phi_1 + |\phi_2| < 1 \text{ and } |\phi_1| < 1 \\ 0 & \text{otherwise} \end{cases}, \tag{3}$$

which leads to a truncated multivariate t density for the marginal posterior of (ϕ_1, ϕ_2).

Figure 1 shows the average and 95% intervals for the estimated posterior mean of $\phi_1 + \phi_2$ values from $M = 300$ simulated dataset for each specification. First, sum of the AR parameters are overestimated except for the correctly specified model with $p(\kappa) = 0$. Hence when the structural breaks in the mean is not taken into account, estimated persistence in the series is higher than the true value. Second, when the issue of structural breaks is more severe, corresponding to relatively high

$p(\kappa)$ values, overestimation in persistence is also more severe and the estimated mean of $\phi_1 + \phi_2$ is close to the boundaries of the parameter space, $\phi_1 + \phi_2 = 1$. Finally, if the true parameters lie at the boundary of the parameter space $\phi_1 + \phi_2 = 1$, the sum of the parameters are underestimated, even for the correctly specified model. We conjecture that this stems from the violation of the stationarity assumption for the model in (2).

-Insert Figure 1 about here-

We next illustrate the overestimation of persistence parameters and its effect on the forecast error using simulated data from (1) with $\phi_2 = 0$. The misspecified model is then given in (2) with the restriction $\phi_2 = 0$. Figure 2 shows the average and 95% intervals for the estimated posterior mean of ϕ_1 from $M = 300$ simulated dataset for different persistence and structural break probabilities. Similar to the AR(2) model with structural breaks, persistence is overestimated under the misspecified model. This overestimation is more severe if the expected number of breaks are higher, with relatively high $p(\kappa)$ values. Furthermore, the size of overestimation in this AR(1) model increases with the true value of the persistence parameter, as shown in the right-panel of Figure 2.

-Insert Figure 2 about here-

We finally consider the effect of such overestimated persistence on the model's forecast performance. We simulate 200 in-sample observations and 100 out-of-sample observations from (1) with different persistence levels and different numbers of (expected) structural breaks. In all simulations, the following are the true parameters: $\phi_1 = 0.1, \sigma_1^2 = 4, \sigma_2^2 = 1, c_0 = 1, \nu_0 = \nu_{-1} = 2$. Notice that the misspecified model, not accounting for structural breaks in (1) will underestimate the data if there is a positive shock to the long-run mean, $\eta_{1,t} > 0$, and overestimate the data in the opposite case $\eta_{1,t} < 0$. If this over and underestimation properties are simply averaged, the net effect of misspecification on the forecast performance cannot be spotted. We therefore distinguish positive and negative long-run mean shocks in the forecast sample: For each simulated data specification, two forecast samples are created such that the shocks to the long-run mean in the forecast sample are positive and negative, respectively. The left and right panels in Figure 3 correspond to these positive and negative jumps in the long-run mean, respectively. For the correctly specified model with $p(\kappa) = 0$, average probability of overestimation in both cases is around 0.5, the expected value for a correctly specified model. The effect of misspecification is visible in both cases, and this effect is more severe if the expected number of structural breaks are higher, with high values of $p(\kappa)$ or if the AR parameter ϕ_2 is relatively high given ϕ_1, i.e. when the true persistence in the data is high.

-Insert Figure 3 about here-

We conclude that the misspecification of the shifts in the long-run mean will alter the estimation and forecast results substantially. This effect is more severe if the expected number of breaks in the long-run mean and the persistence in the data is more severe. Generalizing from the univariate models considered in this section, we next show that NKPC model estimates may also suffer from such misspecification in the behavior of the inflation and marginal cost series.

Our next consideration is the effect of such misspecification on the Bayesian inference of a multivariate model, namely the standard NKPC model. In section 3 we discuss the main issues in Bayesian estimation of the NKPC model. In section 4 we extend the standard NKPC model to allow for level changes over time.

3 Standard NKPC model

We first summarize the structural and reduced form representations of the standard New Keynesian Philips Curve (NKPC). Furthermore, we show that the prior densities adapted for these models should be very carefully chosen due a highly nonlinear transformation of parameters between the structural and reduced form representations.

The standard NKPC with a Calvo formulation is (Calvo, 1983; Galí and Gertler, 1999):

$$\begin{aligned} p_t &= \psi p_{t-1} + (1-\psi)p_t^*, \\ p_t^* &= (1-\gamma\psi)\textstyle\sum_{k=0}^{\infty}(\gamma\psi)^k E_t(z_{t+k}), \end{aligned} \tag{4}$$

where p_t and z_t denote the price level and a proxy variable for real marginal cost at time t, $\psi \in [0,1]$ is the Calvo parameter indicating the weight firms allocate to previous price level in comparison to the expected future price level p_t^*, and $\gamma \in [0,1]$ is the discount rate for expected future marginal cost.

The structural form (SF) representation for the NKPC model derived from (4) is:

$$\begin{aligned} \pi_t &= \lambda z_t + \gamma E(\pi_{t+1}) + \epsilon_{1,t}, \\ z_t &= \phi_1 z_{t-1} + \phi_2 z_{t-2} + \epsilon_{2,t}, \end{aligned} \tag{5}$$

where $(\epsilon_{1,t}, \epsilon_{2,t})' \sim NID\left(0, \left(\begin{smallmatrix} \sigma_{\epsilon_1}^2 & \rho \\ \rho & \sigma_{\epsilon_2}^2 \end{smallmatrix}\right)\right)$ and the unobserved variable $E(\pi_{t+1})$ can be derived as a function of the past marginal cost values z_{t-1} and z_{t-2}, and standard stationary restrictions should hold for ϕ_1, ϕ_2.

The corresponding unrestricted reduced form (URF) representation for (5) is:

$$\begin{aligned} \pi_t &= \alpha_1 z_{t-1} + \alpha_2 z_{t-2} + \varepsilon_t, \\ z_t &= \phi_1 z_{t-1} + \phi_2 z_{t-2} + \epsilon_{2,t} \end{aligned} \tag{6}$$

where $(\varepsilon_t, \epsilon_{2,t})' \sim NID\left(\begin{smallmatrix} \sigma_{\varepsilon}^2 & \rho \\ \rho & \sigma_{\epsilon_2}^2 \end{smallmatrix}\right)$, and the restricted reduced form (RRF) representation is obtained by introducing the following restrictions on parameters in (5):

$$\alpha_1 \quad = \tfrac{\lambda(\phi_1+\gamma\phi_2)}{1-\gamma(\phi_1+\gamma\phi_2)}, \;\; \alpha_2 = \tfrac{\lambda\phi_2}{1-\gamma(\phi_1+\gamma\phi_2)}, \tag{7}$$

Finally, the model in (5) is related to an Instrumental Variables (IV) model with exact identification. Appendix A provides the calculation of the reduced form representation and the relation of the NKPC model with standard IV models.

Bayesian estimation of the unrestricted reduced form model in (6) is straightforward under flat or conjugate priors. Given the posterior draws from these parameters, posterior draws structural form parameters in (5) can be obtained using the transformation in (7). This nonlinear transformation, however, causes difficulties in setting the priors in an adequate way. The determinant of the Jacobian of this nonlinear transformation is[1]:

$$\mid J \mid = \frac{\lambda\phi_2^2}{(1-\gamma(\phi_1+\gamma\phi_2))^2}, \tag{8}$$

where the Jacobian non-zero and finite if: $\gamma(\phi_1 + \gamma\phi_2) \neq 1$, $\phi_2 \neq 0$ and $\lambda \neq 0$. Condition $\phi_2 \neq 0$ is simply the system of equations in (6). Note that the model can also be linked to an exactly identified instrumental variables (IV) model where z_{t-2} is the instrument. Hence the instrument in this model has no explanatory power, and the model is not identified if this condition does not hold. Even if the model is identified, as $\phi_2 \to 0$, the probability mass in the unreasonable regions of the structural parameters increases dramatically, which is to the weak identification problem.

Figure 4 illustrates the nonlinear transformation for the SF and RRF representations, where we get a grid of parameter values from SF representations, and plot the corresponding RRF parameter values, and vice versa. The top panel in Figure 4

[1]We only consider the transformation from $\{\lambda, \gamma, \phi_1, \phi_2\}$ to $\{\alpha_1, \alpha_2, \phi_1, \phi_2\}$, i.e. variance parameters in the transformed model are left as free parameters.

shows the transformations from SF to RRF. Reduced form parameters α_1 and α_2 tend to infinity when persistence in inflation and marginal cost series are high, i.e. when the structural form parameters λ and $\phi_1 + \phi_2$ tend to 1. The bottom panel in Figure 4 shows the RRF to SF transformations. The corresponding SF parameters lead to an irregular shape, for example if the instrument z_{t-2} has no explanatory power with $\phi_2 = 0$ or if $\alpha_2 = 0$.

-Insert Figure 4 about here-

We conjecture that due to the highly nonlinear transformation of parameters between the SF and RRF representations, a non-informative prior for the RRF may lead to highly informative priors for the SF. If the analysis aims at inferring the SF parameters, the results may be very sensitive to the choice of the prior for the reduced form parameters. As an example, consider the following informative priors for the reduced form parameters:

$$\phi_1 \sim N(0.6, 0.1),\ \phi_2 \sim N(-0.04, 0.1) \tag{9}$$

$$\alpha_1 \sim N(0.02, 0.005),\ \alpha_2 \sim N(-0.025, 0.001), \tag{10}$$

where the corresponding density of the SF parameters are given in Figure 5. These rather 'tight' priors on reduced form parameters and the implied 'wide' priors on structural form parameters as the structural form parameters, such as the Calvo parameter, are bounded by definition.

-Insert Figure 5 about here-

We note that the NKPC analysis outlined in this section requires demeaned and detrended inflation and marginal cost series, π_t and z_t, respectively. In the next section we discuss the effect of such demeaning and detrending in detail. As outlined in section 2, this analysis can be inaccurate since the level and trend behavior in the series is assumed to be constant over time. In the next section we extend the standard NKPC model to account for possible changes in levels and trends.

4 An extended NKPC model

4.1 NKPC model with raw data

The standard practice in estimating the NKPC model is to demean and detrend the series a priori. The specific methodology used for detrending the series can severely bias the results, and fail to account for the structure of the time series, see Gorodnichenko and Ng (2010); Canova (2012). To overcome this issue, we consider the raw series, without demeaning or detrending, and combine structural time series techniques with the structural form resulting from NKPC. Effectively, we estimate the level and trend of the marginal cost and inflation series along with NKPC model parameters.

For the empirical analysis, we consider US inflation and real marginal cost series over the period from 1960 first quarter until 2011 fourth quarter. Inflation is computed as the growth rate of implicit GDP deflator and labor share in non-farm business sector.[2] The raw series of US inflation and labour share, and a preliminary indication of changing levels in the series are displayed in Figure 6. This crude analysis shows that the levels of these series change substantially over time. Hence simply demeaning and detrending these series may deteriorate the inference on NKPC parameters.

-Insert Figure 6 about here-

From the top panel in Figure 6, we observe two distinct periods with differing patterns for the raw inflation series. The period between the beginning of 1970s and

[2] http://research.stlouisfed.org/fred2/

beginning of 1980s can be labeled as high inflationary period compared to remaining periods. Existing evidence show that the decline in the level and volatility is due to credible monetary policy that stabilized inflationary expectations at a low level via commitment to a nominal anchor since the early eighties, see McConnell and Perez-Quiros (2000); Stock and Watson (2002); Ahmed et al. (2004); Stock and Watson (2007); Cecchetti et al. (2007). One way to model this changing behavior of the series to allow for regime changes in parameters to capture the change in the structure of the series, see Cogley and Sargent (2005); Sims and Zha (2006); Kim and Nelson (2006); Canova and Gambetti (2006), among others. We also include the level (unconditional mean) of the inflation series in the upper panel of the Figure 6 with structural breaks in the fourth quarter of 1967 and the first quarter of 1983 in line with the existing findings.[3] The figure indicates a temporary increase in the level of inflation during 1970s, while this increase in the inflation switches back to the earlier levels after the second break in the first quarter of 1983. We, therefore, model the level of the inflation allowing for permanent level shifts incorporating the changes in the level of inflation in the NKPC. Let denote the level of the inflation in period t as $c_{\pi,t}$, this corresponds to the following model

$$c_{\pi,t+1} = c_{\pi,t} + \kappa_t \eta_{1,t+1} \tag{11}$$

where κ_t is a binary variable taking the value of 1 with probability p_κ if there is level shift and 0 $1 - p_\kappa$ if the level does not change and $\eta_{1,t} \sim N(0, \sigma^2_{\eta_1})$. This model structure allows for occasional level shifts depending on the probability p_κ of the binomial process preserving a parsimonious model structure with only a single additional parameter. The magnitude of the level changes is determined by the

[3]This pattern does not change with the marginal changes in terms of the timing of the breaks, which correspond to the period where the Federal Reserve Board reserve-targeting policies had been replaced with the interest rate targeting policy rule. Moreover, Cecchetti et al. (2007), among other papers, point another shift in the level of inflation around late 1960s as the start of the high inflationary period.

parameter σ_η, where occasional and large level shifts corresponds to low values of $1 - p_\kappa$ together with relatively high values of σ_η and the opposite case corresponds to the local level model, see Giordani et al. (2007) for a similar approach.

Second, when labor share series is analyzed in the bottom panel of Figure 6, unlike inflation series we do not observe discrete changes during the course of time. Instead, the labor share series exhibits a continuously changing pattern due to a negative trend. As from the figure this trend is more prominent in the second half of the sample period, we allow for a changing trend using a local linear trend specification as follows

$$\begin{aligned} c_{z,t+1} &= \mu_{z,t} + c_{z,t} + \eta_{2,t+1} \\ \mu_{z,t+1} &= \mu_{z,t} + \eta_{3,t+1} \end{aligned} \qquad (12)$$

where $\eta_{2,t} \sim N(0, \sigma^2_{\eta_2})$ and $\eta_{3,t} \sim N(0, \sigma^2_{\eta_3})$, see Durbin and Koopman (2001) for details. This specification is flexible enough encompassing many types of filters used for detrending including Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997) employed prior to estimation of the NKPC model, see Canova (2012) for a similar specification in the more general context of DSGE models. When $\sigma^2_{\eta_3} = 0$, for example, the level of the labor share follows a random walk with a drift, μ_z. Additionally, when $\sigma^2_{\eta_2} = 0$, a deterministic trend is obtained. On the other hand, setting only $\sigma^2_{\eta_2} = 0$ but allowing $\sigma^2_{\eta_3}$ to be positive results in an integrated random walk trend which can approximate many types of nonlinear trends including HP filter and the parameters of the HP filter can be recovered under certain re-parametrization, see Harvey and Jaeger (1993); Harvey and Trimbur (2008); Harvey (2011). Moreover, Delle Monache and Harvey (2011) show the robustness of the (12) against many types of model misspecification.

Together with the level specifications of the inflation and labor share series the

NKPC takes the following form

$$\begin{aligned}
\pi_t - c_{\pi,t} &= \lambda\left(z_t - c_{z,t}\right) + \gamma E(\pi_{t+1} - c_{\pi,t+1} \mid I_t) + \epsilon_{1,t} \\
z_t - c_{z,t} &= \phi_1\left(z_{t-1} - c_{z,t-1}\right) + \phi_2\left(z_{t-2} - c_{z,t-2}\right) + \epsilon_{2,t} \\
c_{\pi,t+1} &= c_{\pi,t} + \kappa_t \eta_{1,t+1} \\
c_{z,t+1} &= \mu_{z,t} + c_{z,t} + \eta_{2,t+1} \\
\mu_{z,t+1} &= \mu_{z,t} + \eta_{3,t+1}
\end{aligned}, \tag{13}$$

where $\begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{\epsilon_1} & \rho\sigma_{\epsilon_1}\sigma_{\epsilon_2} & 0 & 0 & 0 \\ \rho\sigma_{\epsilon_1}\sigma_{\epsilon_2} & \sigma^2_{\epsilon_2} & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\eta_1} & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\eta_2} & 0 \\ 0 & 0 & 0 & & \sigma^2_{\eta_3} \end{pmatrix} \right)$

Since the measurement equation is written still in demeaned and detrended form, the reduced form of the NKPC in section 3, equation (6) still holds. In other words, the transformation functions and the Jacobian related to the change in the measurement remain identical except that the time varying parameters are replaced with the corresponding constant parameters.

A further refinement in the NKPC model can be achieved allowing for time dependency in residual variances. This extension is particularly appealing for the inflation series, as the variance of this series seems to change over time substantially, see e.g. Stock and Watson (2008) for similar model with a stochastic volatility component. To extend the NKPC model with a stochastic volatility process in the inflation shocks, we set the following state equation to the system

$$h_{t+1} = h_t + \eta_{4,t+1},\ \eta_{4,t+1} \sim NID(0, \sigma^2_{\eta_4}), \tag{14}$$

where $\sigma^2_{\epsilon_{1,t}} = \exp(h_t/2)$.

5 Bayesian inference

The reduced form of the NKPC model in (13) can be written in state-space form as follows

$$\begin{aligned} Y_t &= HX_t + BU_t + \epsilon_{\mathbf{t}} \quad \epsilon_{\mathbf{t}} \sim N(0, Q_t) \\ X_t &= FX_{t-1} + R_t \eta_{\mathbf{t}} \qquad \eta_{\mathbf{t}} \sim N(0, I) \end{aligned} \tag{15}$$

where

$$Y_t = \begin{pmatrix} \pi_t \\ z_t \end{pmatrix} \quad X_t = \begin{pmatrix} c_{\pi,t} \\ c_{z,t} \\ \mu_{z,t} \\ c_{z,t-1} \\ c_{z,t-2} \end{pmatrix} \quad U_t = \begin{pmatrix} z_{t-1} \\ z_{t-2} \end{pmatrix} \quad \epsilon_{\mathbf{t}} = \begin{pmatrix} \varepsilon_t \\ \epsilon_t \end{pmatrix}$$

$$H = \begin{pmatrix} 1 & 0 & 0 & -\alpha_1 & -\alpha_2 \\ 0 & 1 & 0 & -\phi_1 & -\phi_2 \end{pmatrix} \quad B = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \phi_1 & \phi_2 \end{pmatrix} \quad Q_t = \begin{pmatrix} \sigma^2_{\epsilon_1,t} & \rho\sigma_{\epsilon_1,t}\sigma_{\epsilon_2} \\ \rho\sigma_{\epsilon_1,t}\sigma_{\epsilon_2} & \sigma^2_{\epsilon_2} \end{pmatrix}$$

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad R_t = \begin{pmatrix} \kappa_t\sigma_{\eta_1} & 0 & 0 \\ 0 & \sigma_{\eta_2} & 0 \\ 0 & 0 & \sigma_{\eta_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \eta_{\mathbf{t}} = \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \end{pmatrix}$$

Once the state-space form of the model is set as in (15), Bayesian inference can be carried on by computing posterior distributions for the extended NKPC model. These are obtained by combining the prior specifications together with the likelihood functions. We specify prior distribution and the resulting simulation scheme in the next section. The resulting posterior distributions are given in Appendix B.

5.1 Prior specifications

The NKPC model and the extensions we propose aim to infer the unrestricted reduced form parameters, and sample the structural parameters of the NKPC structural parameters in return. As outlined in section 3, inference on the structural parameters can be very sensitive to the prior choice on the reduced form parameters. Specifically, tight priors on reduced form parameters can be quite uninformative for the structural parameters, and vice versa.

We choose conjugate priors for the unrestricted reduced form parameters, which lead to uninformative priors for the structural form parameters of the NKPC:

$$(\phi_1, \phi_2, \alpha_1, \alpha_2)' \sim N\left((0.6,\ -0.04,\ 0.02,\ -0.025)', \text{diag}(0.1,\ 0.1,\ 0.005,\ 0.001)\right), \tag{16}$$

where diag(a) corresponds to the identity matrix with the diagonal elements replaced by the vector a. The effect of these parameters on the structural NKPC parameters and the corresponding Calvo parameter are provided in section 3.

We also motivate this prior choice using a prior predictive analysis. The prior predictive analysis is based on the priors in (16), together with an Inverse Wishart distribution for the covariance parameters: $\Sigma = \left(\begin{smallmatrix} \sigma_{\epsilon^2_{1,t}} & \rho\sigma_{\epsilon^2_{1,t}}\sigma_{\epsilon_2} \\ \rho\sigma_{\epsilon^2_{1,t}}\sigma_{\epsilon_2} & \sigma_{\epsilon^2_2} \end{smallmatrix}\right) \sim IW\left(\left(\begin{smallmatrix} 0.35 & 0.1 \\ 0.1 & 2.5 \end{smallmatrix}\right) \times 10, 10\right)$ for $t = 1, \ldots, T$, where $IW(A, b)$ is the Inverse Wishart distribution with scale matrix A and degrees of freedom b. We simulate $10,000$ parameter values from this prior specification and simulate $10,000$ data points from the NKPC model for each of these parameter values. If the average sample moments of these simulated data sets, namely the 'implied sample moments', are close to the sample moments of the observed data, the prior specification is said to be accurate for the data. Figure 7 shows these implied sample moments, together with the real data moments. The implied sample moments from the prior specification we have are substantially close to the true data moments. Hence the priors on the unrestricted reduced form parameters follow the data pattern accurately. In the remaining of the applications, priors for the NKPC model are hence based on (16).

-Insert Figure 7 about here-

For the extended NKPC models, the prior specification is more involved than the standard NKPC model. Together with the prior on the structural parameters in (16), we consider conjugate priors of the inverse Gamma form corresponding to the following specifications:

$$\begin{aligned}
\sigma_{\epsilon_1} \mid \sigma_{\epsilon_2}, \rho &\sim IG\left(\nu_{\epsilon_1}, \Phi_{\epsilon_1}\right), \\
\sigma_{\epsilon_2} \mid \rho &\sim IG\left(\nu_{\epsilon_2}, \Phi_{\epsilon_2}\right), \\
\rho &\sim (1-\rho^2)^{-3/2}, \\
\sigma_{\eta_i} &\sim IG\left(\nu_{\eta_i}, \Phi_{\eta_i}\right), \quad \text{for } i = 1, \ldots, 4.
\end{aligned} \tag{17}$$

Next, we specify the prior distribution for the structural break probability:

$$p_\kappa \mid \sigma, \theta \sim \text{beta}(\delta_1, \delta_2), \tag{18}$$

where beta(.) is the beta distribution with scale parameters δ_1 and δ_2. We choose these parameters such that $E(p_\kappa) = 0.02$, i.e. a priori we expect on average 4 structural breaks for a sample with 200 observations. We impose the informative prior of this form since we want to capture structural breaks that are rare but large in magnitude. Furthermore, we restrict the prior distributions for the URF model, such that the corresponding structural parameters are all in the interval $(-10, 10)$, i.e. the conjugate priors in (16) are truncated in this region.

As discussed in section 2, we note that the prior parameters should be carefully chosen if we aim to infer the structural NKPC parameters from the unrestricted reduced form parameters of this extended model in (13). We discuss this issue in detail in section 6, where we apply the extended NKPC model on US inflation and marginal cost data.

5.2 Posterior simulation

As the number and the location of the structural breaks are unknown the likelihood function is intractable. Instead, we set up an MCMC algorithm to sample from the full conditional posterior distributions. Specifically, we use Gibbs sampling together with data augmentation (see Geman and Geman, 1984; Tanner and Wong, 1987) to obtain posterior results. Let denote $Y_{1:T} = (Y_1, Y_2, \dots, Y_T)'$, $X_{1:T} = (X_1, X_2, \dots, X_T)'$, $U_{1:T} = (U_1, U_2, \dots, U_T)'$, $\sigma^2_{\epsilon_1,1:T} = (\sigma^2_{\epsilon_1,1}, \sigma^2_{\epsilon_1,2}, \dots, \sigma^2_{\epsilon_1,T})'$ and $\theta = (\phi_1, \phi_2, \alpha_1, \alpha_2)'$. The resulting simulation scheme is as follows

1. Initialize the parameters by drawing κ_t using the prior in (18) and unobserved states X_t, h_t for $t = 1, 2, \dots, T$ from standard normal distribution and conditional on κ_t for $t = 0, 1, \dots, T$. At step (m) of the iteration
2. Sample $\theta^{(m)}$ from $p(\theta|Y_{1:T}, X_{1:T}, U_{1:T}, R_{1:T}, Q_{1:T})$.
3. Sample $X_t^{(m)}$ from $p(X_t|\theta^{(m)}, Y_{1:T}, h_{1:T}, U_{1:T}, R_{1:T}, Q_{1:T})$ for $t = 1, 2, \dots, T$.
4. Sample $h_t^{(m)}$ from $p(h_t|X_{1:T}^{(m)}, \theta^{(m)}, Y_{1:T}, X_{1:T}, U_{1:T}, R_{1:T}, \rho^{m-1}, \sigma^{2,(m-1)}_{\epsilon_2}, \sigma^{2,(m-1)}_{\eta_4})$ for $t = 1, 2, \dots, T$.
5. Sample $\kappa_t^{(m)}$ from $p(\kappa^{(m)}|\theta^{(m)}, Y_{1:T}, h_{1:T}, U_{1:T}, R_{1:T}, Q_{1:T})$ for $t = 1, 2, \dots, T$.
6. Sample $p_\kappa^{(m)}$ from $p(p_\kappa^{(m)}|\kappa_{1:T}^{(m)})$.
7. Sample $\sigma^{2,(m)}_{\eta_i}$ from $p(\sigma^{2,(m)}_{\eta_i}|X_{1:T}^{(m)}, h_{1:T}^{(m)}, \kappa_{1:T}^{(m)})$ for $i = 1, 2, 3, 4$.
8. Sample $\rho^{(m)}$ from from $p(\rho^{(m)}|X_{1:T}^{(m)}, h_{1:T}^{(m)}, Y_{1:T}, X_{1:T}, U_{1:T}, \theta^{(m)}, \sigma^{2,(m-1)}_{\epsilon_2})$.
9. Sample $\sigma^{2,(m)}_{\epsilon_2}$ from from $p(\sigma^{2,(m)}_{\epsilon_2}|\rho^{(m)}, X_{1:T}^{(m)}, h_{1:T}^{(m)}, Y_{1:T}, X_{1:T}, U_{1:T}, \theta^{(m)})$.
10. Repeat (2)-(9) M times.

Steps (3)-(6) are common to many models in the Bayesian state-space framework, see for example Kim and Nelson (1999); Gerlach et al. (2000). Details of the full posterior conditional distributions for these parameters are given in Appendix B.

6 Application to US data

In our empirical analysis, we use US inflation and real marginal cost series as outlined in section 4. We first estimate the standard NKPC model by detrending the data prior to estimation. Next, we consider the proposed model where we simultaneously estimate the trends and levels in the series along with other model parameters. For the former, we use two conventional detrending techniques, namely linear detrending and the HP filter. For the proposed model, we first estimate the model in equation 13 by setting the structural break probabilities to 1. This model corresponds to a local level specification for the inflation level and is denoted as 'TVP NKPC'. Second, we estimate the proposed model with occasional level shifts, where the shift probability is not restricted to be 1. This model is denoted as 'TVP-LS NKPC. Estimation results of the standard NKPC model using HP filtered and linear filtered series together with the NKPC model with endogenous detrending, as shown in (13), are given in Table 1 and corresponding posterior distributions are displayed in Figure 8.

-Insert Table 1 about here-

-Insert Figure 8 about here-

First, from the graph and table we can observe the effect of weak identification on the structural parameters. For the structural parameter γ, when we consider the models using detrended data, the corresponding posterior distributions are bimodal. This is due to the fact that the posterior distribution of ϕ_2 has a mass in both positive and negative regions, leading to bimodality of the posterior γ distribution.

In addition to this bimodality, the posterior density of γ is spread in an extremely wide region with most of the probability mass around the boundaries of the prior distribution, -10 and 10, indicating the weak identification of this parameter. A similar result holds for the posterior distribution of the Calvo parameter ψ since the posterior density is concentrated around 0 and 1. Furthermore, although both distributions are bimodal, posterior modes switch dramatically depending on how the low frequency movements of the data are removed a priori: i.e. using the HP filter or linear filter to remove the trend in the marginal cost series. This obscures inference on structural parameters substantially in the sense that the economic interpretation of the results rely heavily on the detrending method.

When we consider the models with endogenous detrending, however, the identification problem in the structural parameters disappears. The posterior density of the inflation adjustment parameter γ is unimodal and the probability mass is concentrated in a reasonable region, despite its wide support. For the model with level shifts and inflation, shown in panel D of Figure 8, the posterior density is even more concentrated, facilitating inference on this parameter. A similar result holds for the Calvo parameter as well. The bimodality of posterior distribution of Calvo parameter observed in the standard NKPC models with filtered data vanishes. The precision of the parameter increases further when the level shifts in inflation series are taken into account.

Inference on λ parameter is relatively less sensitive to the detrending method compared to parameters γ and ψ. The difference between results for the models with a priori detrending is not as pronounced, despite the apparent non-normality of the distribution when HP filter is employed. Still, the precision improves when the extended models are examined. In particular, posterior λ distribution has no probability mass in the negative region unlike the previous models. Note that if the other parameters are in the positive regions as the economic theory suggests, λ

parameter should be positive as well.

Although the proposed methods mitigate the identification issues and improve the inference for the NKPC structural form parameters, the posterior distribution still supports parameter values outside reasonable regions, implying that the weak identification issues are not completely resolved. One way to mitigate the effects of the weak identification is to further restrict the prior distributions. Such restrictions are non-trivial unless economic theory is incorporated in forming the priors, see Del Negro and Schorfheide (2008) for this class of priors. Indeed, we also resort to economic theory still preserving a general prior structure. First, the theory underlying NKPC assigns γ as a discount factor, which firms use to discount future stream of profits. Therefore, we restrict γ to be in the interval $[0, 1]$ as discount factor cannot be negative and a value exceeding 1 implies that firms overvalue future profits. Second, Calvo parameter, ψ, shows the probability that firms keep the same price level for the next period price setting implying that this parameter has to be restricted in the interval $[0, 1]$. We estimate the models with endogenous detrending imposing these restrictions to alleviate the effects of weak identification and to increase the precision of economic interpretation. Estimation results of the standard NKPC model with endogenous detrending together with these restrictions, as shown in (13), is given in Table 2 and corresponding posterior distributions are displayed in Figure 9.

-Insert Table 2 about here-

-Insert Figure 9 about here-

Table 2 and Figure 9 show that, compared to the previous results in Table 1 and Figure 8, precision of the structural parameters are improved dramatically. This implies that the restrictions we impose facilitate inference of the structural parameters.

Posterior mean of the γ parameter is slightly lower than the values reported in the literature, which indicate values around 0.9 see Galí et al. (2001); Nason and Smith (2008). Notice that parameter γ is the parameter which relates inflation to expected future inflation. This implies that higher inflation persistence is reflected in higher values of γ. The relatively lower values of γ around 0.7 relates to our discussion in section 2, where we show that misspecified low frequency moments in the data may yield to estimated persistence levels which are higher than the true value. Since both proposed models account for changes in low frequency movements using structural time series models, the estimated persistence of inflation is lower which corresponds to lower values of γ. When we incorporate structural breaks in the level of inflation explicitly, as in equation (13), the decrease in the estimated γ parameter is more pronounced. This shows the importance of incorporating level shifts in inflation in the NKPC model.

The effect of modeling trends and levels in the series endogenously is also seen in the posterior distribution of λ and ψ. Posterior mean of λ is slightly higher than the values reported in the literature indicating values around 0.3 see Galí et al. (2001); Nason and Smith (2008). According to our results the sensitivity of inflation to real marginal cost is even more pronounced with a higher precision. This sensitivity increases further when we incorporate structural breaks explicitly, as shown in the second column of Table 2. Our results on the price stickiness parameter ψ, with a posterior mean of 0.82 in Table 2, are in line with the findings in the literature, with values around 0.85 (Galí et al., 2001). This implies that, on average, 82% of the firms do not adjust their price level.

Apart from the NKPC model parameters, the time series structure of the inflation and marginal cost series are also of interest. Since we explicitly model unobserved levels of inflation and marginal cost series, estimates of these parameters are also obtained along with other model parameters. We display the evolution of these estimated levels over time in Figure 10. Panel A and B of the figure shows the estimates for the TVP NKPC model and the TVP-LS NKPC model, respectively.

-Insert Figure 10 about here-

The top figures in both panels in Figure 10 show the estimated inflation levels over the period from the second quarter of 1960 to the last quarter of 2011. In both models, there is a clear time variation in inflation levels. Estimated levels from the TVP NKPC model indicate an increasing inflation level during 1970s and the beginning of 1980s. This period is documented as a high inflationary period, see Cecchetti et al. (2007); Harvey (2011), which is nicely captured by the time varying structure of the model. When we impose occasional breaks using the TVP-LS NKPC model the estimates of the time varying inflation level become much smoother, as shown in the bottom panel. The inflation periods are captured more precisely under this model structure in the sense that the model locates two distinct periods with high and low inflation levels. The high inflationary period covers the period between late 1960s and early 1980s. After this period, however, the inflation level is almost anchored around the value 0.5, corresponding to the annual inflation rate around 2% (Cecchetti et al., 2007; Giannone et al., 2008). From our results, it seems that changing the monetary policy to a nominal target anchors the long-run inflation expectations, i.e. time-varying inflation levels in our setting.

The middle and bottom figures in both panels of Figure 10 show the estimated level and trend of the marginal cost series. Similar to the inflation series, the level of the marginal cost series is also changing substantially over time. While marginal cost levels fluctuate around a constant level until the end of 1990s, this pattern seems to change after 2000. After this time marginal cost level tends to decrease. This is also seen in the bottom figures in panels A and B of Figure 10. The slope of the marginal cost trend in these figures increase in absolute value over time with a negative sign causing a downward slope in the marginal cost level.

(Results of the Extended NKPC model with stochastic volatility to be added)

(Results of the forecasting exercise to be added)

7 Conclusion

The NKPC model constitutes an integral part of macroeconomic models used for policy analysis. These models are mostly estimated after demeaning and/or detrending the series. This mechanical removal of the low frequency movements in the data, however, may lead to misspecification plaguing inference and causing issues such as weak identification of the model parameters. Potential structural breaks and level shifts in the observed series require more complex models, which can handle these time variation together with the standard NKPC parameters. We propose such a model where we infer the low and high frequency movements in the inflation and marginal cost series jointly. This is achieved by modeling the levels and trends of the series explicitly in the NKPC model and estimating these along with other model parameters simultaneously.

The proposed model captures time variation in the low frequency moments of both inflation and marginal cost data. Incorporating such time variation in the model improves the precision of estimated model parameters and mitigate the issue of weak identification in the standard NKPC model. Furthermore, modeling level changes explicitly decreases estimated persistence in inflation substantially compared to those obtained by estimating the NKPC model using a priori demeaned and detrended series. In terms of the low frequency moments in the series, estimated inflation levels identify two distinct periods with high and low inflation. This corresponds to the high inflationary period during the 1970s and the beginning of 1980s. The period starting with 1980s is characterized by low inflation levels corresponding to an annual inflation level around 2%. Similarly, estimated marginal cost levels are subject to fluctuations with a clear downward trend after 2000.

The framework proposed in this paper can be extended in a number of ways. In terms of the econometric methodology, inference can be based on the structural parameters directly. Such a methodology leads to highly irregular and intractable posterior distributions, requiring complex simulation methods. In terms of the economic content, several extensions of the NKPC model, such as the hybrid Phillips curve, can be incorporated in our framework.

Appendix A Reduced Form and Instrumental Variables representations of the NKPC model

The structural form (SF) representation for the NKPC model is:

$$\pi_t = \lambda z_t + \gamma E(\pi_{t+1}) + \epsilon_{1,t}, \tag{A.1}$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \epsilon_{2,t}, \tag{A.2}$$

where $(\epsilon_{1,t}, \epsilon_{2,t})' \sim NID\left(0, \begin{pmatrix} \sigma^2_{\epsilon_1} & \rho \\ \rho & \sigma^2_{\epsilon_2} \end{pmatrix}\right)$.

Note that the future inflation expectation in (A.1) is unobserved, hence is obtained by iterating (A.1):

$$E(\pi_{t+1}) = \lambda E(z_{t+1}) + \gamma E(\pi_{t+2}), \tag{A.3}$$

where $E(.)$ denotes the expectation given information at time t.

Inserting (A.2) and (A.3) in (A.1) yields:

$$\pi_t = \lambda z_t + \lambda \sum_{k=1}^{\infty} \gamma^k E(z_{t+k}) + \epsilon_{1,t}, \tag{A.4}$$

$$= \lambda \sum_{k=0}^{\infty} \gamma^k E(z_{t+k}) + \epsilon_{1,t}, \tag{A.5}$$

where we use the equality $z_t = E(z_t)$.

Inserting (A.2) and (A.3) in (A.5), and rearranging the results, we have:

$$\pi_t = \phi_1 \lambda \sum_{k=0}^{\infty} \gamma^k E(z_{t+k-1}) + \phi_2 \lambda \sum_{k=0}^{\infty} \gamma^k E(z_{t+k-2}) + \epsilon_{1,t}, \tag{A.6}$$

$$= (\phi_1 + \phi_2\gamma)\lambda z_{t-1} + \phi_2 \lambda z_{t-2} + (\phi_1 + \phi_2\gamma)\lambda\gamma \sum_{k=0}^{\infty} \gamma^k E(z_{t+k}) + \epsilon_{1,t}, \tag{A.7}$$

We replace the expectation term in (A.7) by (A.5):

$$\lambda \sum_{k=0}^{\infty} \gamma^k E(z_{t+k}) = \pi_t - \epsilon_{1,t}, \tag{A.8}$$

The restricted (and unrestricted) reduced form for the NKPC model is hence:

$$\pi_t = \underbrace{\frac{(\phi_1 + \phi_2\gamma)\lambda}{1-(\phi_1+\phi_2\gamma)\gamma}}_{\alpha_1} z_{t-1} + \underbrace{\frac{\phi_2\lambda}{1-(\phi_1+\phi_2\gamma)\gamma}}_{\alpha_2} z_{t-2} + \epsilon_{1,t} \tag{A.9}$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \epsilon_{2,t}, \tag{A.10}$$

where $(\epsilon_{1,t}, \epsilon_{2,t})' \sim NID\left(0, \left(\begin{smallmatrix} \sigma^2_{\epsilon_1} & \rho \\ \rho & \sigma^2_{\epsilon_2} \end{smallmatrix}\right)\right)$.

It is also straightforward to show that the NKPC model relates to an exactly identified Instrumental Variables (IV) model. Inserting (A.3) in (A.4) we get:

$$\pi_t = (1 + (\phi_1 + \phi_2\gamma)\gamma)\lambda z_t + \phi_2\lambda\gamma z_{t-1} + (\phi_1+\phi_2\gamma)\lambda\gamma^2 \sum_{k=0}^{\infty} \gamma^k E(z_{t+k+1}) + \epsilon_{1,t}, \tag{A.11}$$

We replace the expectation term in (A.11) by (A.4):

$$\lambda\gamma \sum_{k=0}^{\infty} \gamma^k E(z_{t+k+1}) = \pi_t - \lambda z_t - \epsilon_{1,t}, \tag{A.12}$$

Hence the NKPC model is identical to an exactly identified IV model:

$$\pi_t = \underbrace{\frac{\lambda}{1-(\phi_1+\phi_2\gamma)\gamma}}_{\alpha_1^{iv}} z_t + \underbrace{\frac{\phi_2\lambda\gamma}{1-(\phi_1+\phi_2\gamma)\gamma}}_{\alpha_2^{iv}} z_{t-1} + \epsilon_{1,t}. \tag{A.13}$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + \epsilon_{2,t}, \tag{A.14}$$

where parameters $\alpha_1^{iv}, \alpha_2^{iv}$ are IV model's the structural form parameters, and inference on these parameters is not possible under flat priors, due to exact identification (Zellner et al., 1988).

Appendix B Bayesian inference of the extended NKPC model

In this appendix, we derive the posterior distributions that we use in the sampling scheme described in Section 5.2. Let us recall the system in the state-space form

$$\begin{aligned} Y_t &= HX_t + BU_t + \epsilon_{\mathbf{t}} \quad \epsilon_{\mathbf{t}} \sim N(0, Q_t) \\ X_t &= FX_{t-1} + R_t \eta_{\mathbf{t}} \qquad \eta_{\mathbf{t}} \sim N(0, I) \end{aligned} \tag{B.15}$$

B.1 Sampling of θ

Conditional on the states $c_{\mu,t}, c_{z,t}$ and h_t for $t = 0, 2, \ldots, T$, redefining the variables such that

$$\begin{aligned} Z_t &= (y_t - c_{\mu,t}, z_t - c_{z,t}) && \text{for } t = 2, \cdots, T \\ D_t &= (z_{t-1} - c_{z,t-1}, z_{t-2} - c_{z,t-2}) && \text{for } t = 2, \cdots, T, \end{aligned} \tag{B.16}$$

the measurement equation in (B.15) can be rewritten as

$$Z_t = BD_t + \epsilon_{\mathbf{t}} \text{ and } \epsilon_{\mathbf{t}} \sim N(0, Q_t). \tag{B.17}$$

In compact form the model becomes

$$Z = BD + U, \tag{B.18}$$

where $Z = (z_1, z_2, \cdots, z_T)$, $D = (D_1, D_2, \cdots, D_T)$ and $U = (\epsilon_1, \epsilon_2, \cdots, \epsilon_T)$. Using the fact that $\text{vec}(AB) = (B' \otimes I)\text{vec}(A)$, where $\bigotimes$ stands for the Kronecker product, the following univariate model can be written

$$\begin{aligned} \text{vec}(Z) &= (D' \otimes I_4) * \theta + vec(U) \\ z &= (D' \otimes I_4) * \theta + u \\ \text{where Cov}(u) &= \Omega = \text{diag}(Q_1, Q_2, \cdots, Q_T), \end{aligned} \tag{B.19}$$

Using the multivariate normal prior $N(\underline{\mu}_\theta, \underline{\Sigma}_\theta)$ in (16) the posterior distribution is Gaussian, $\theta|z \sim N(\overline{\mu}_\theta, \overline{\Sigma}_\theta)$, with the following parameters

$$\begin{aligned} \overline{\Sigma}_\theta &= \left(\underline{\Sigma}_\theta^{-1} + (D' \otimes I_4)'\Omega^{-1}(D' \otimes I_4)\right)^{-1} \\ \overline{\mu}_\theta &= \overline{\Sigma}_\theta \left(\underline{\Sigma}_\theta^{-1}\underline{\mu}_\theta + (D' \otimes I_4)'\Omega^{-1}(D' \otimes I_4)\theta_{OLS}\right) \\ \text{where } \theta_{OLS} &= ((D' \otimes I_4)'\Omega^{-1}(D' \otimes I_4))^{-1}(D' \otimes I_4)'\Omega^{-1}z. \end{aligned} \tag{B.20}$$

B.2 Sampling of states, X_t

Conditional on the remaining model parameters, drawing $X_{0:T}$ can be implemented using standard Bayesian inference. This constitutes running the Kalman filter first and running a simulation smoother using the filtered values for drawing smoothed states as in Carter and Kohn (1994) and Frühwirth-Schnatter (1994). In the first step, start the recursion for $t = 1, \ldots, T$

$$\begin{aligned} X_{t|t-1} &= FX_{t-1|t-1} \\ P_{t|t-1} &= FP_{t-1|t-1}F' + R_t'R_t \\ \eta_{t|t-1} &= y_t - HX_{t|t-1} - BU_t \\ \zeta_{t|t-1} &= HP_{t|t-1}H' + Q_t \\ K_t &= P_{t|t-1}H'\zeta_{t|t-1}' \\ X_{t|t} &= X_{t|t-1} + K_t\eta_{t|t-1} \\ P_{t|t} &= P_{t|t-1} - K_tH'\zeta_{t|t-1}', \end{aligned} \tag{B.21}$$

and store $X_{t|t}$ and $P_{t|t}$. The last filtered state $f_{T|T}$ and its covariance matrix $P_{T|T}$ correspond to the smoothed estimates of the mean and the covariance matrix of the factors for period T. Having stored all the filtered values, simulation smoother involves the following backward recursions for $t = T-1, \ldots, 1$

$$
\begin{aligned}
\eta^*_{t+1|t} &= X_{t+1} - FX_{t|t} \\
\zeta^*_{t+1|t} &= FP_{t|t}F' + R'_{t+1}R_{t+1} \\
X_{t|t,X_{t+1}} &= X_{t|t} + P_{t|t}F'\zeta^{*-1}_{t+1|t}\eta^*_{t+1|t} \\
P_{t|t,P_{t+1}} &= P_{t|t} - P_{t|t}F'\zeta^{*-1}_{t+1|t}FP_{t|t}.
\end{aligned}
\tag{B.22}
$$

Intuitively, the simulation smoother updates the states using the same principle as in the Kalman filter, where at each step filtered values are updated using the smoothed values obtained from backward recursion. For updating the initial states, using the state equation $X_{0|t,X_1} = F^{-1}(f_1)$ and $P_{0|t,P_1} = F^{-1}(P_1 + R'_1R_1)F'^{-1}$ can be written for the first observation. Given the mean $X_{t|t,X_{t+1}}$ and the covariance matrix $P_{t|t,P_{t+1}}$, the states can be sampled from $X_t \sim N(X_{t|t,X_{t+1}}, P_{t|t,P_{t+1}})$ for $t = 0, ..., T$.

B.3 Sampling of inflation volatilities, h_t

Conditional on the remaining model parameters, drawing $h_{0:T}$ can be implemented using standard Bayesian inference as in the case of X_t. One important difference, however, stems from the logarithmic transformation of the variance in (14). As the transformation concerns the error structure, the square of which follows a χ^2 distribution, the system is not Gaussian but logarithm of χ^2 distribution. Noticing the properties of logarithm of χ^2 distribution Kim et al. (1998) and Omori et al. (2007) approximate this distribution using mixture of Gaussian distributions. Hence, conditional on these mixture components the system remains Gaussian allowing for standard inference as in section B.2. We do not explain the procedure in detail but refer to Omori et al. (2007) for details.

B.4 Sampling of structural break parameters, κ_t

Sampling of structural break parameters, κ_t, can be implemented by computing the posterior for the binary outcomes, i.e. the posterior values in case of structural break in period t and the posterior value of the case of no structural breaks. However, evaluating this posterior requires one sweep, which is of order $O(T)$. As this evaluation should be implemented for each period t the resulting procedure would be of order $O(T^2)$. When the number of sample size is large this would result in an infeasible scheme. One way to by pass this problem and to obtain an algorithm of order $O(T)$ is to condition on the estimated states, X_t but this may cause the chain to break down completely with the increasing correlation between the structural break parameters and the states. Fortunately, Gerlach et al. (2000) propose an efficient algorithm for sampling structural break parameters, κ_t, conditional on the observed data, which is still of order $O(T)$. We also implement this algorithm for estimation of the structural breaks and refer to Gerlach et al. (2000); Giordani and Kohn (2008) for details.

B.5 Sampling of structural break probability, p_κ

Conditional on $\kappa_{1:T}$, the remaining model parameters do not provide additional information for the sampling of the structural break probability. Therefore, the combined with the prior distribution in (18) the resulting posterior distribution becomes a beta distribution with the following parameters, beta$(\delta_1 + k, \delta_2 + T - k)$, where k is the number of structural breaks with $\kappa_t = 1$ and T is the sample size.

B.6 Sampling of state error variances, σ_η^2

Using standard results from a linear regression model with a conjugate prior for the variance in (17), it follows that the conditional posterior distribution of $\sigma_{\eta_i}^2$, with $i = 1, 2, 3, 4$ is an inverted χ^2 distribution with scale parameter $\Phi_{\eta_i} + \sum_{t=1}^{T} \eta_{i,t}^2$ and with $T + \nu_{\eta_i}$ degrees of freedom for $i = 2, 3, 4$. For $i = 1$ the parameters of the inverted χ^2 distribution becomes $\Phi_{\eta_1} + \sum_{t=1}^{T} \kappa_t \eta_{1,t}^2$ and $k + \nu_{\eta_1}$.

B.7 Sampling of marginal cost volatility and correlation

To sample the variance of marginal cost and correlation we decompose the multivariate normal distribution of $\epsilon_{\mathbf{t}}$ into the conditional distribution of $\epsilon_{2,t}$ given $\epsilon_{1,t}$ and the marginal distribution of $\epsilon_{1,t}$, as in Çakmaklı et al. (2011). This results in

$$\prod_{t=1}^{T} f(\epsilon_{\mathbf{t}}) = \prod_{t=1}^{T} \frac{1}{\sigma_{\epsilon_{1,t}}} \phi\left(\frac{\epsilon_{1,t}}{\sigma_{\epsilon_{1,t}}}\right) \frac{1}{\sigma_{\epsilon_{2,t}}\sqrt{(1-\rho^2)}} \phi\left(\frac{\epsilon_{2,t} - \rho\epsilon_{1,t}}{\sigma_{\epsilon_{2,t}}(1-\rho^2)}\right), \tag{B.23}$$

Hence, together with prior for the variance in (17), variance of the marginal cost series can be sampled using (B.23) by setting up a Metropolis-Hasting step (Metropolis et al., 1953; Hastings, 1970) using an inverted χ^2 candidate density with scale parameter $\sum_{t=1}^{T} \epsilon_{2,t}^2$ and with T degrees of freedom.

To sample ρ from its conditional posterior distribution we can again use (B.23). Conditional on the remaining parameters the posterior becomes

$$(1-\rho^2)^{-\frac{3}{2}} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{(1-\rho^2)}} \phi\left(\frac{\epsilon_{2,t} - \rho\epsilon_{1,t}}{\sigma_{\epsilon_{2,t}}(1-\rho^2)}\right)\right). \tag{B.24}$$

We can easily implement the griddy Gibbs sampler approach of Ritter and Tanner (1992). Given that $\rho \in (-1, 1)$ we can setup a grid in this interval based on the precision we desire about the value of ρ.

References

Ahmed, S., Levin, A., Wilson, B. A., 08 2004. Recent u.s. macroeconomic stability: Good policies, good practices, or good luck? The Review of Economics and Statistics 86 (3), 824–832.

Bianchi, F., 2010. Regime switches, agents' beliefs, and Post-World War II U.S. macroeconomic dynamics. Working Papers 10–39, Duke University, Department of Economics.

Calvo, G., September 1983. Staggered prices in a utility-maximizing framework. Journal of monetary Economics 12 (3), 383–398.

Canova, F., May 2012. Bridging dsge models and the raw data. Working Papers 635, Barcelona Graduate School of Economics.

Canova, F., Gambetti, L., Jan. 2006. Structural changes in the us economy: Bad luck or bad policy? CEPR Discussion Papers 5457, C.E.P.R. Discussion Papers.

Carter, C. K., Kohn, R., 1994. On gibbs sampling for state space models. Biometrika 81 (3), 541–553.

Çakmaklı, C., Paap, R., Van Dijk, D., 2011. Modeling and estimation of synchronization in multistate Markov-switching models. Working Papers 11-002/4, Tinbergen Institute.

Cecchetti, S. G., Hooper, P., Bruce C. Kasman, K. L. S., Watson, M. W., 2007. Understanding the evolving inflation process. Tech. rep., paper written for the U.S. Monetary Policy Forum 2007, Washington, DC.

Clarida, R., Galí, J., Gertler, M., February 2000. Monetary policy rules and macroeconomic stability: Evidence and some theory. The Quarterly Journal of Economics 115 (1), 147–180.

Cogley, T., Sargent, T. J., April 2005. Drift and volatilities: Monetary policies and outcomes in the post wwii u.s. Review of Economic Dynamics 8 (2), 262–302.

DeJong, D. N., Dave, C., 2011. Structural macroeconometrics. Princeton University Press.

Del Negro, M., Schorfheide, F., October 2008. Forming priors for DSGE models (and how it affects the assessment of nominal rigidities). Journal of Monetary Economics 55 (7), 1191–1208.

Delle Monache, D., Harvey, A. C., 2011. The effect of misspecification in models for extracting trends and cycles. Tech. rep.

Durbin, J., Koopman, S. J., August 2001. Time Series Analysis by State Space Methods. OUP Catalogue. Oxford University Press.

Frühwirth-Schnatter, S., 1994. Data augmentation and dynamic linear models. Journal of Time Series Analysis 15, 183–202.

Galí, J., Gertler, M., 1999. Inflation dynamics: A structural econometric analysis. Journal of monetary Economics 44 (2), 195–222.

Galí, J., Gertler, M., Lopez-Salido, J. D., 2001. European inflation dynamics. European Economic Review 45 (7), 1237–1270.

Geman, S., Geman, D., 1984. Stochastic relaxations, Gibbs distributions, and the bayesian restoration of images. IEEE Transaction on Pattern Analysis and Machine Intelligence 6, 721–741.

Gerlach, R., Carter, C., Kohn, R., 2000. Efficient bayesian inference for dynamic mixture models. Journal of the American Statistical Association, 819–828.

Giannone, D., Lenza, M., Reichlin, L., 04–05 2008. Explaining the Great Moderation: It is not the shocks. Journal of the European Economic Association 6 (2-3), 621–633.
URL http://ideas.repec.org/a/tpr/jeurec/v6y2008i2-3p621-633.html

Giordani, P., Kohn, R., January 2008. Efficient bayesian inference for multiple change-point and mixture innovation models. Journal of Business & Economic Statistics 26, 66–77.

Giordani, P., Kohn, R., van Dijk, D., March 2007. A unified approach to nonlinearity, structural change, and outliers. Journal of Econometrics 137 (1), 112–133.

Gorodnichenko, Y., Ng, S., April 2010. Estimation of DSGE models when the data are persistent. Journal of Monetary Economics 57 (3), 325–340.

Harvey, A., 2011. Modelling the Phillips curve with unobserved components. Applied Financial Economics 21 (1-2), 7–17.

Harvey, A. C., Jaeger, A., 1993. Detrending, stylized facts and the business cycle. Journal of Applied Econometrics 8 (3), 231–47.

Harvey, A. C., Trimbur, T., 2008. Trend estimation and the hodrick-prescott filter. Journal of Japanese Statistical Society (38), 41–9, in honour of H. Akaike.

Hastings, W. K., 1970. Monte carlo sampling using Markov chains and their applications. Biometrika 57, 97–10.

Hodrick, R. J., Prescott, E. C., February 1997. Postwar U.S. business cycles: An empirical investigation. Journal of Money, Credit and Banking 29 (1), 1–16.

Kim, C.-J., Nelson, C. R., 1999. State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications, 1st Edition. Vol. 1. The MIT Press.

Kim, C.-J., Nelson, C. R., November 2006. Estimation of a forward-looking monetary policy rule: A time-varying parameter model using ex post data. Journal of Monetary Economics 53 (8), 1949–1966.

Kim, S., Shephard, N., Chib, S., July 1998. Stochastic volatility: Likelihood inference and comparison with arch models. Review of Economic Studies 65 (3), 361–93.

Kleibergen, F., Mavroeidis, S., 2009. Weak instrument robust tests in GMM and the New Keynesian Phillips Curve. Journal of Business and Economic Statistics 27 (3), 293–311.

Kleibergen, F., Mavroeidis, S., 2011. Identification robust priors for Bayesian analysis in DSGE models. working paper.

Mavroeidis, S., 2004. Weak identification of forward-looking models in monetary economics*. Oxford Bulletin of Economics and Statistics 66, 609–635.

Mavroeidis, S., 2005. Identification issues in forward-looking models estimated by GMM, with an application to the Phillips Curve. Journal of Money, Credit and Banking 37 (3), 421–48.

McConnell, M. M., Perez-Quiros, G., December 2000. Output fluctuations in the United States: What has changed since the early 1980's? American Economic Review 90 (5), 1464–1476.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., Teller, E., 1953. Equations of state calculations by fast computing machines. Journal of Chemical Physics 21, 1087–1092.

Nason, J., Smith, G., 2008. Identifying the New Keynesian Phillips curve. Journal of Applied Econometrics 23 (5), 525–551.

Omori, Y., Chib, S., Shephard, N., Nakajima, J., October 2007. Stochastic volatility with leverage: Fast and efficient likelihood inference. Journal of Econometrics 140 (2), 425–449.

Ritter, C., Tanner, M. A., 1992. Facilitating the Gibbs sampler: The gibbs stopper and the griddy-Gibbs sampler. Journal of the American Statistical Association 87 (419), 861–868.

Sims, C. A., Zha, T., March 2006. Were there regime switches in U.S. monetary policy? American Economic Review 96 (1), 54–81.

Stock, J. H., Watson, M. W., Aug. 2002. Has the business cycle changed and why? NBER Working Papers 9127, National Bureau of Economic Research, Inc.

Stock, J. H., Watson, M. W., 02 2007. Why has u.s. inflation become harder to forecast? Journal of Money, Credit and Banking 39 (s1), 3–33.

Stock, J. H., Watson, M. W., September 2008. Phillips curve inflation forecasts. Working Paper 14322, National Bureau of Economic Research.

Tanner, M. A., Wong, W. H., 1987. The calculation of posterior distributions by data augmentation. Journal of the American Statistical Association 82, 528–550.

Zellner, A., Bauwens, L., Van Dijk, H. K., 1988. Bayesian specification analysis and estimation of simultaneous equation models using monte carlo methods. Journal of Econometrics 38 (1-2), 39–72.

Zhang, C., Osborn, D. R., Kim, D. H., 2008. The new Keynesian Phillips curve: from sticky inflation to sticky prices. Journal of Money, Credit and Banking 40 (4), 667–699.

Tables and Figures

Table 1: Estimation results of NKPC models

	HP NKPC	LT NKPC	TVP NKPC	TVP-LS NKPC
γ	-4.226 (7.335)	6.973 (4.179)	2.817 (3.484)	2.588 (3.105)
λ	0.035 (0.047)	0.045 (0.037)	0.125 (0.003)	0.124 (0.004)
ψ	0.778 (0.377)	0.183 (0.211)	0.432 (0.043)	0.459 (0.045)

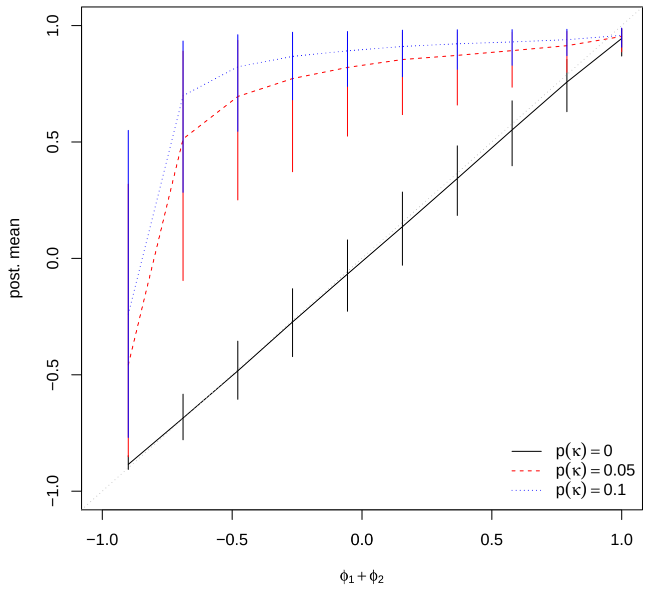
Note: The table presents estimation results of structural parameters from different types of NKPC models. The models are applied to quarterly inflation and real marginal cost series for the sample period over first quarter 1960 and fourth quarter 2011. 'HP NKPC' is the model where the NKPC model is estimated using detrended series using HP filter. 'LT NKPC' is the model where the NKPC model is estimated using detrended series using linear filter. 'TVP NKPC' is the model where the NKPC model is estimated using original series using the model in (13) where the level of the inflation is modeled using local level model by setting the break probabilities to 1. 'TVP-LS NKPC' is the model where the NKPC model is estimated using original series using the model in (13) where the level of the inflation is modeled using local level model together with level shifts as in (13). γ is the coefficient of the next period inflation expectation and λ is the coefficient of real marginal cost. ψ is the Calvo parameter determining the degree of price stickiness. Posterior results are based on 20,000 simulations of which the first 10,000 are discarded as burn-in sample. The convergence of the MCMC sampler is checked using statistical and visual inspection and in all model specifications convergence is assured.

Table 2: Estimation results of NKPC models with endogenous detrending using further restrictions

	TVP NKPC	TVP-LS NKPC
γ	0.716 (0.049)	0.709 (0.051)
λ	0.088 (0.000)	0.090 (0.000)
ψ	0.817 (0.003)	0.816 (0.003)

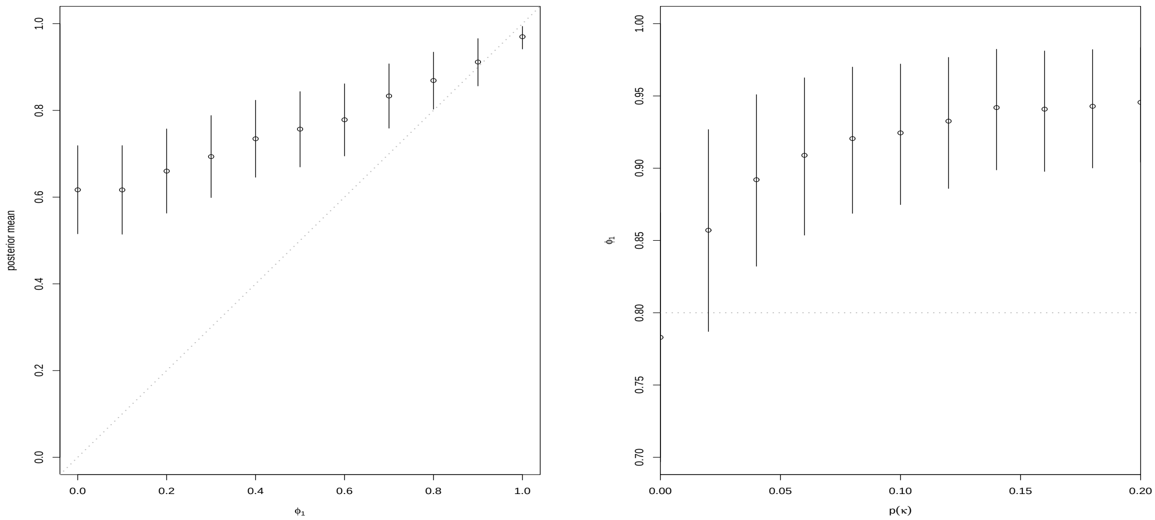
Note: The table presents estimation results of structural parameters from different types of NKPC models. The models are applied to quarterly inflation and real marginal cost series for the sample period over first quarter 1960 and fourth quarter 2011. 'TVP NKPC' is the model where the NKPC model is estimated using original series using the model in (13) where the level of the inflation is modeled using local level model by setting the break probabilities to 1. 'TVP-LS NKPC' is the model where the NKPC model is estimated using original series using the model in (13) where the level of the inflation is modeled using local level model together with level shifts as in (13). γ is the coefficient of the next period inflation expectation and λ is the coefficient of real marginal cost. ψ is the Calvo parameter determining the degree of price stickiness. Posterior results are based on 20,000 simulations of which the first 10,000 are discarded as burn-in sample. The convergence of the MCMC sampler is checked using statistical and visual inspection and in all model specifications convergence is assured.

Figure 1: Overestimation illustration for a misspecified AR(2) model with structural breaks in the mean in the long-run mean



Note: The figure presents overestimation probability for simulated data with different persistence levels and different (expected) numbers of structural breaks in the mean. The data are simulated from the AR(2) model with structural breaks in (1). We report average and 95% intervals for estimated posterior mean of $\phi_1 + \phi_2$ based on 300 simulation replications for each parameter setting. The rest of the model parameters are set as $\sigma_1^2 = 4, \sigma_2^2 = 1, c_0 = 1, \nu_0 = \nu_{-1} = 2$.

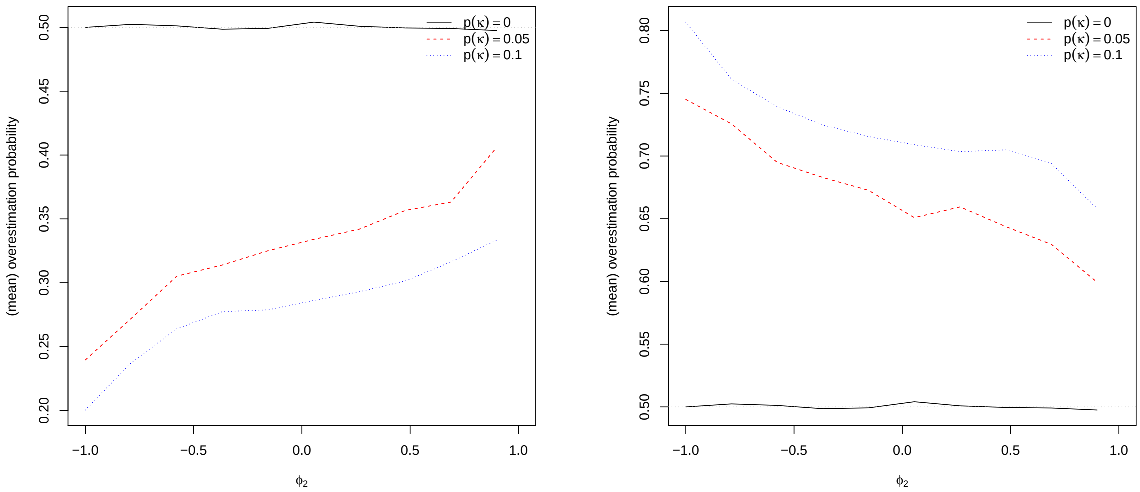
Figure 2: Overestimation illustration for a misspecified AR(1) model with structural breaks in the long-run mean



$p(\kappa) = 0.025$ $\phi = 0.8$

Note: The figure presents overestimation probability for simulated data with different persistence levels and different (expected) numbers of structural breaks in the mean. The data are simulated from the AR(1) model with structural breaks in (1) with $\phi_2 = 0$. We report average and 95% intervals for estimated posterior mean of ϕ_1 based on 300 simulation replications for each parameter setting. The rest of the model parameters are set as $\sigma_1^2 = 4, \sigma_2^2 = 1, c_0 = 1, \nu_0 = \nu_{-1} = 2$.

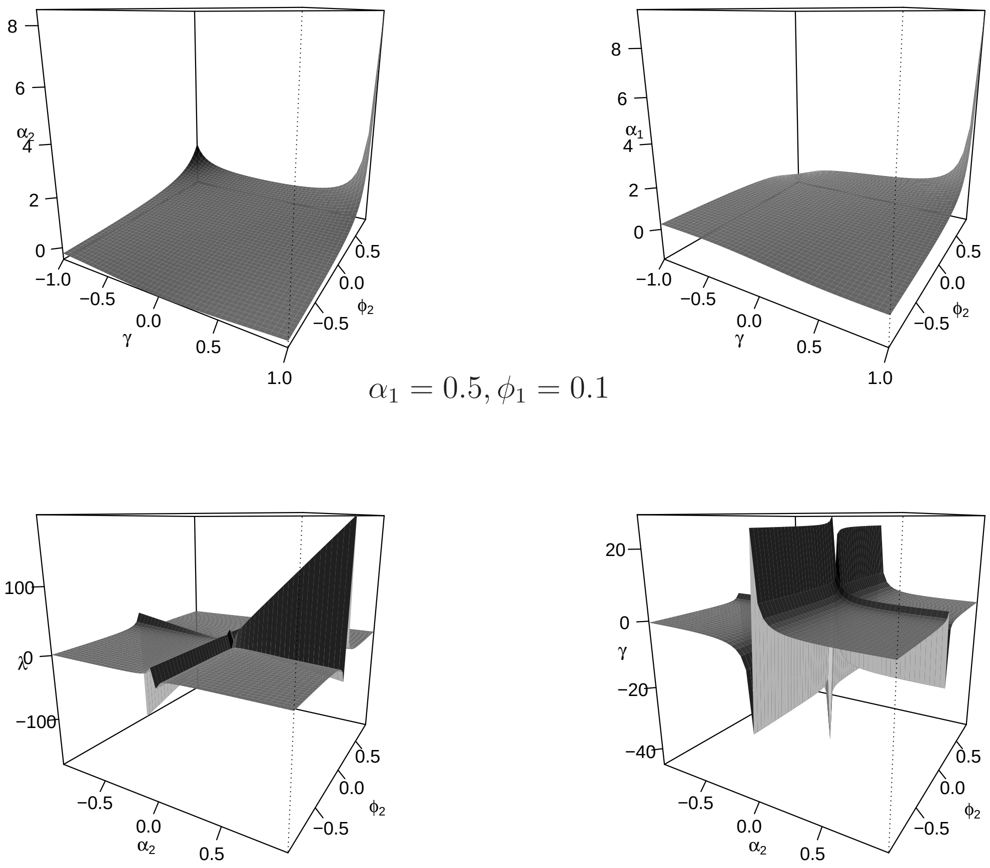
Figure 3: Forecast performance illustration for a misspecified AR(2) model with structural breaks in the long-run mean



positive jumps in forecast sample　　　　negative jumps in forecast sample

Note: The figure presents overestimation probability for simulated data with different persistence levels and different (expected) numbers of structural breaks in the mean. The data are simulated from the AR(2) model with structural breaks in (1). The rest of the model parameters are set as $\phi_1 = 0.1, \sigma_1^2 = 4, \sigma_2^2 = 1, c_0 = 1, \nu_0 = \nu_{-1} = 2$. The results are based on 300 simulation replications for each parameter setting. The left and right panels show the overestimation probability in the forecast sample, for observations with positive and negative shocks, respectively.

Figure 4: Illustration of the nonlinear parameter transformation from structural form to reduced form parameters (top panel) and reduced form to structural form parameters (bottom panel)

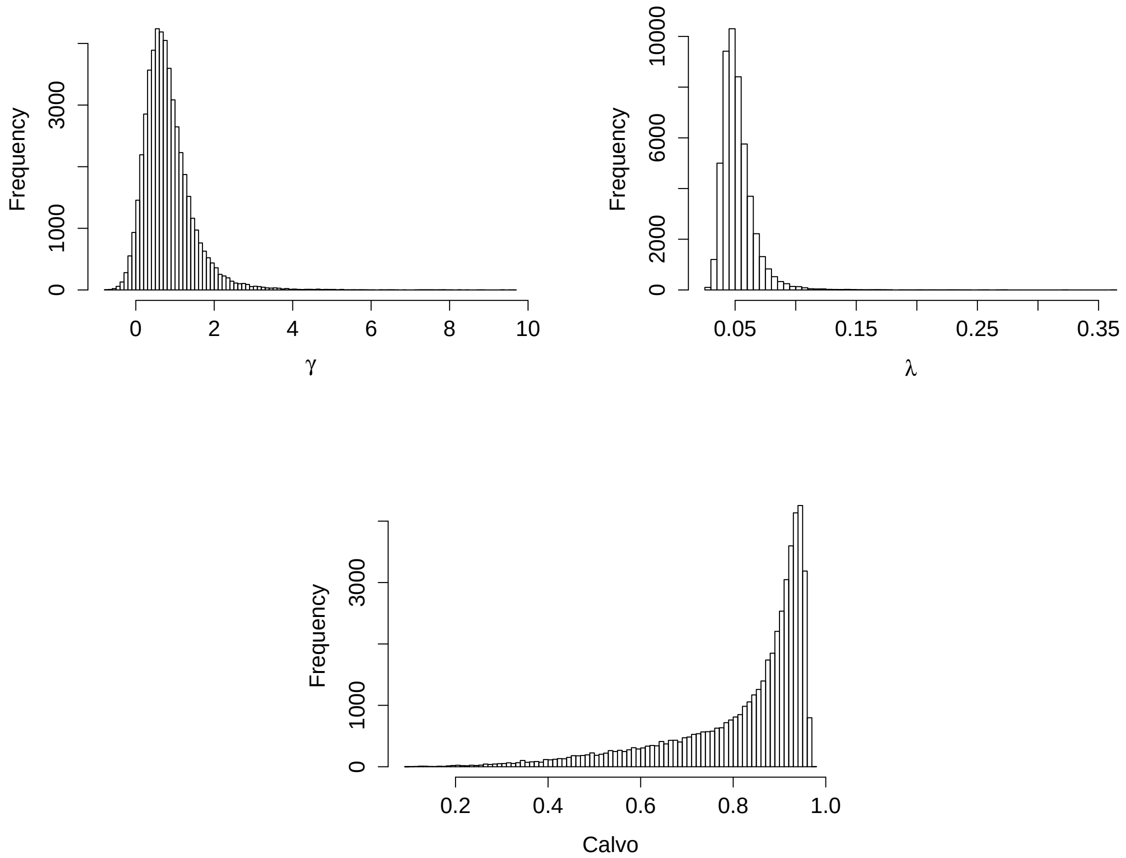


$\alpha_1 = 0.5, \phi_1 = 0.1$

$\lambda = 0.5, \phi_1 = 0.1$

Note: The top panel presents the implied unrestricted reduced form parameters in (6) given structural form parameters in (5). The top panel presents implied structural form parameters in (5) given unrestricted reduced form parameters in (6). Parameter transformations are obtained using the RRF restrictions in (7).

Figure 5: Illustration of implied priors on structural form parameters)



$\phi_1 \sim N(0.6, 0.1)$ and $\phi_2 \sim N(-0.04, 0.1)$ in stationary region,
$\alpha_1 \sim N(0.02, 0.005)$ and $\alpha_2 \sim N(-0.025, 0.001)$

Note: The figure presents implied structural form parameters given unrestricted reduced form priors in (16). γ is the coefficient of the next period inflation expectation and λ is the coefficient of real marginal cost. Calvo parameter, ψ, determines the degree of price stickiness.

Figure 6: Inflation and labor share series over first quarter 1960 and fourth quarter 2011

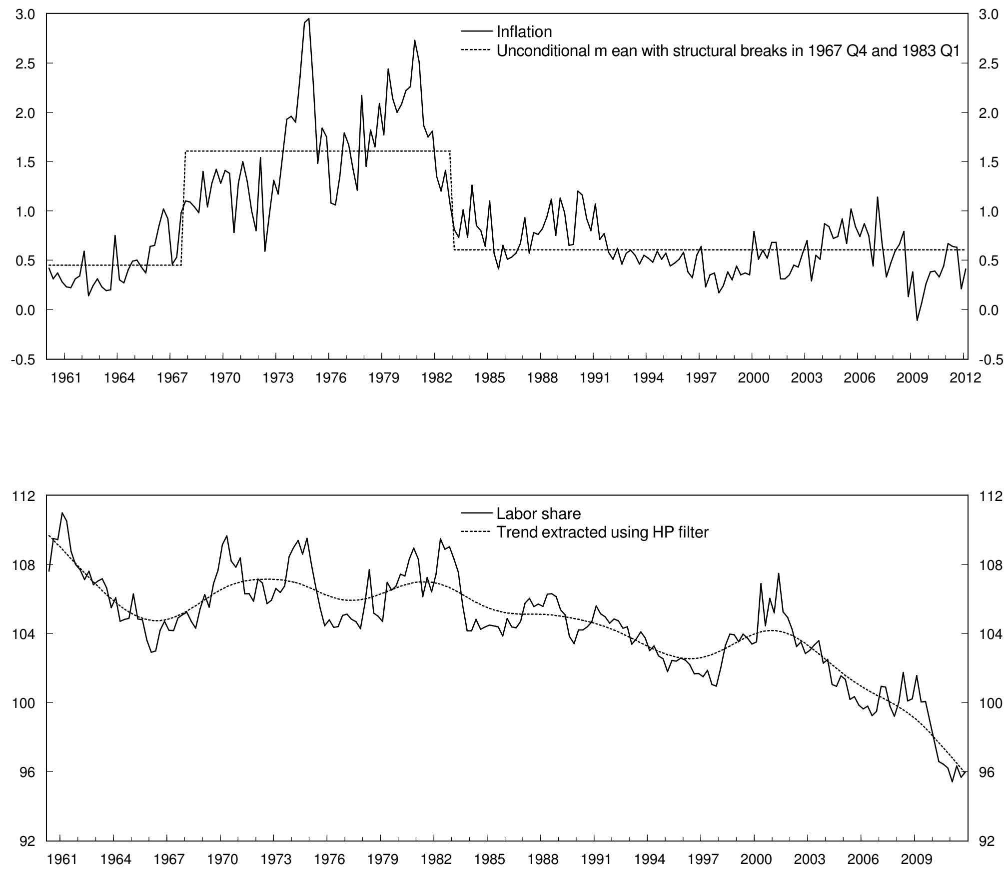
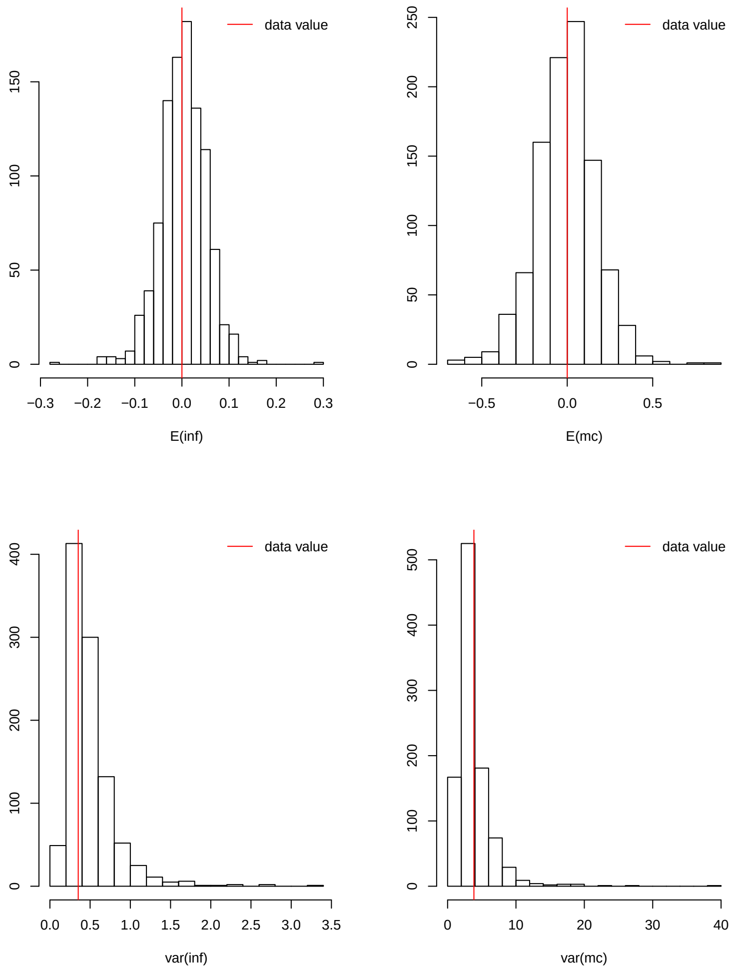


Figure 7: Prior predictive analysis: Implied and true sample moments from the priors on NKPC URF parameters

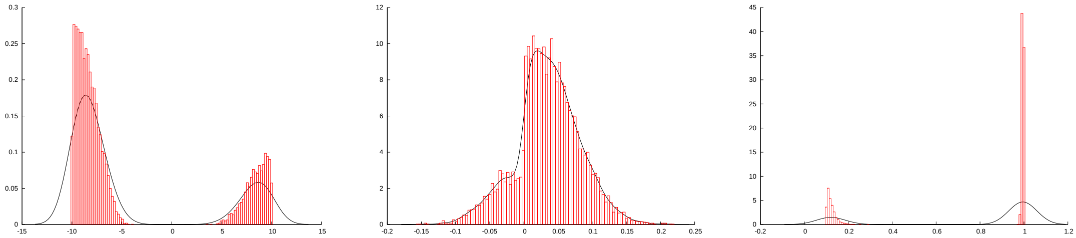


Note: The figure presents implied data moments given the prior specifications. 'E(inf)' and 'E(mc)' denote the implied expectations of inflation and marginal cost series. 'var(inf)' and 'var(mc)' denote the implied variances of the inflation marginal cost series. The vertical lines in all figures are the true data moments. The prior predictive analysis is based on the URF priors in (16) and an Inverse Wishart prior for the covariance parameters: $\Sigma = \begin{pmatrix} \sigma_{\epsilon^2_{1,t}} & \rho\sigma_{\epsilon^2_{1,t}}\sigma_{\epsilon_2} \\ \rho\sigma_{\epsilon^2_{1,t}}\sigma_{\epsilon_2} & \sigma_{\epsilon^2_2} \end{pmatrix} \sim IW\left(\left(\begin{smallmatrix} 0.35 & 0.1 \\ 0.1 & 2.5 \end{smallmatrix}\right) \times 10, 10\right)$ for $t = 1, \ldots, T$. The implied sample moments based on $10,000$ simulations from this prior distribution, and $10,000$ data simulations from the NKPC model and each prior simulation.

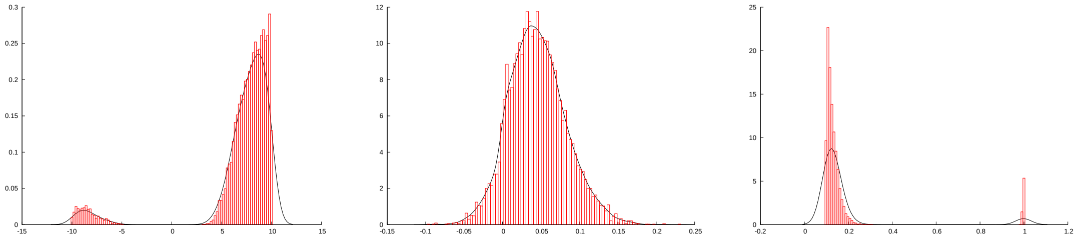
Figure 8: Posterior distributions of structural parameters estimated using NKPC models.

γ	λ	ψ

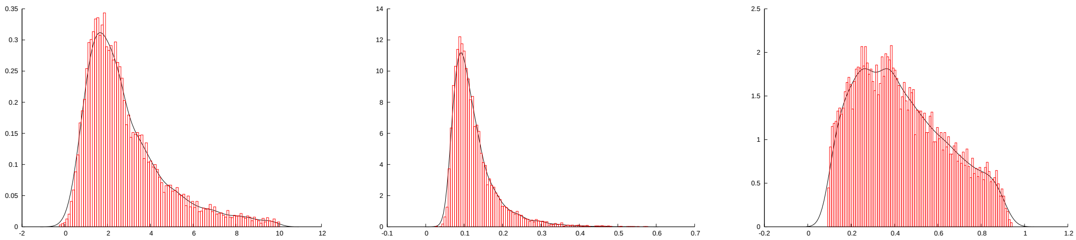
Panel A: HP NKPC



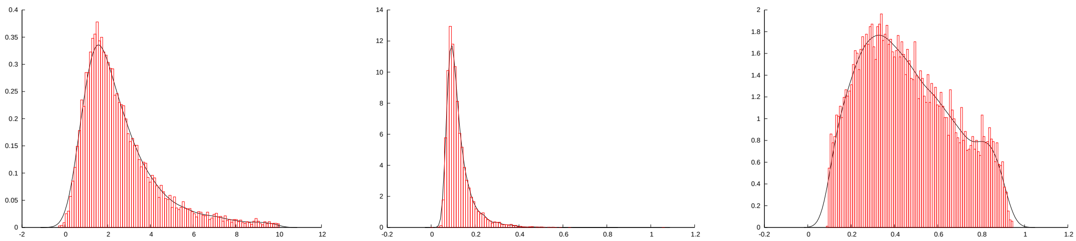
Panel B: LT NKPC



Panel C: TVP NKPC



Panel D: TVP-LS NKPC

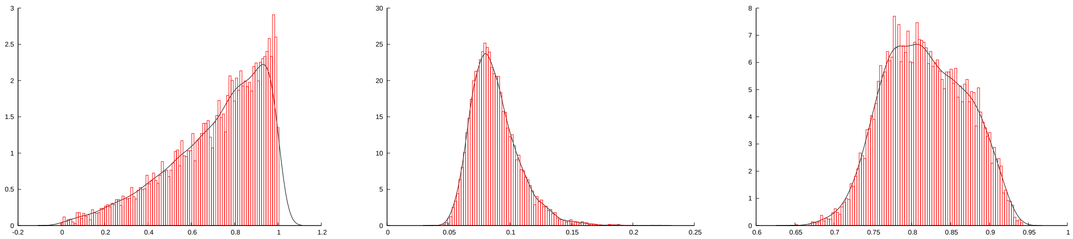


Note: The figure presents posterior distribution of structural parameters from different types of NKPC models. The models are applied to quarterly inflation and real marginal cost series for the sample period over first quarter 1960 and fourth quarter 2011. 'HP NKPC' is the model where the NKPC model is estimated using detrended series using HP filter. 'LT NKPC' is the model where the NKPC model is estimated using detrended series using linear filter. 'TVP NKPC' is the model where the NKPC model is estimated using original series using the model in (13) where the level of the inflation is modeled using local level model by setting the break probabilities to 1. 'TVP-LS NKPC' is the model where the NKPC model is estimated using original series using the model in (13) where the level of the inflation is modeled using local level model together with level shifts as in (13). γ is the coefficient of the next period inflation expectation and λ is the coefficient of real marginal cost. ψ is the Calvo parameter determining the degree of price stickiness. Posterior results are based on 20,000 simulations of which the first 10,000 are discarded as burn-in sample. The convergence of the MCMC sampler is checked using statistical and visual inspection and in all model specifications convergence is assured.

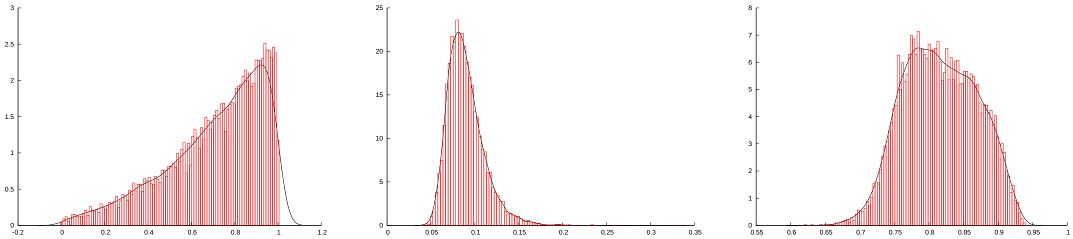
Figure 9: Posterior distributions of structural parameters estimated using NKPC models.

γ	λ	ψ

Panel A: TVP NKPC



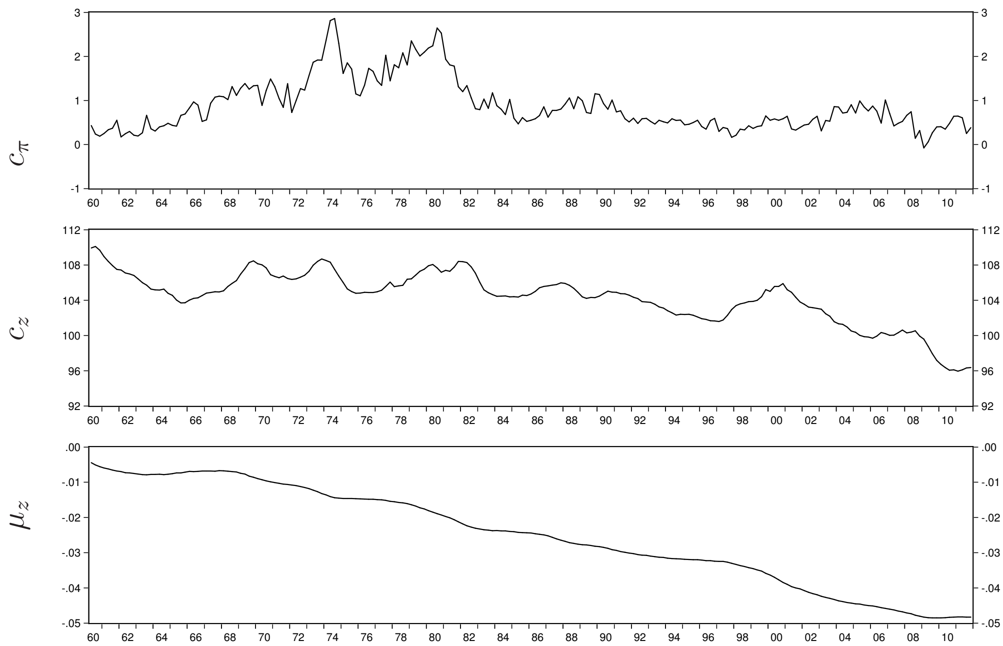
Panel B: TVP-LS NKPC



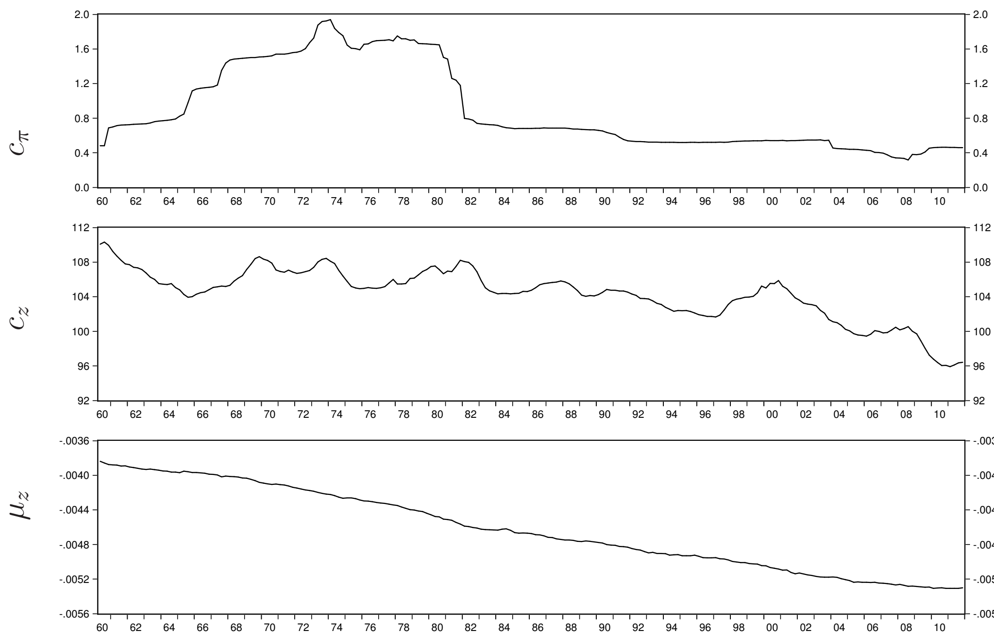
Note: The figure presents posterior distribution of structural parameters from different types of NKPC models. The models are applied to quarterly inflation and real marginal cost series for the sample period over first quarter 1960 and fourth quarter 2011. 'TVP NKPC' is the model where the NKPC model is estimated using original series using the model in (13) where the level of the inflation is modeled using local level model by setting the break probabilities to 1. 'TVP-LS NKPC' is the model where the NKPC model is estimated using original series using the model in (13) where the level of the inflation is modeled using local level model together with level shifts as in (13). γ is the coefficient of the next period inflation expectation and λ is the coefficient of real marginal cost. ψ is the Calvo parameter determining the degree of price stickiness. Posterior results are based on 20,000 simulations of which the first 10,000 are discarded as burn-in sample. The convergence of the MCMC sampler is checked using statistical and visual inspection and in all model specifications convergence is assured.

Figure 10: Posterior means of time varying parameters in NKPC models.

Panel A: TVP NKPC



Panel B: TVP-LS NKPC



Note: The figure presents posterior means of time varying parameters (states) in NKPC models. The top figures in both panels are the evolution of the inflation level over time. The middle figures are the evolution of the marginal cost level over time. The bottom figures show the time-varying slope of the linear trend in the marginal cost series. See Table 1 for abbreviations.