Evidence on Features and Sources of EMU Business Cycles using Bayesian Panel Markov-switching VAR

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Abstract

This paper proposes a panel Markov-Switching (MS-) VAR model suitable for a multi-country analysis of the business cycle. We study the business cycles fluctuations of a group of countries, analyse the transmission of shocks across cycles allowing for heterogeneity and asymmetries among countries, and predict the turning points of the country-specific cycles. We focus on the European Monetary Union (EMU) and compare the results obtained by analysing the EMU at a disaggregated level. We propose a forecast combination approach for aggregating the turning points of the EMU countries in order to obtain a possibly better prediction of the turning points for the EMU business cycle and understand its sources. A Bayesian approach has been applied to estimate the panel MS-VAR model and to forecast the turning points.

1   Introduction

In this paper, we contribute to the literature on the analysis of the business cycle of large panel of countries. The analysis of the world business cycle has been proposed by Gregory et al. (1997), who consider a panel of trivariate series (output, consumption and investment) for the G7 countries and estimate dynamic factor model featuring a common (world) cycle, a country specific component and a series specific (fully idiosyncratic) one. The specification of the model is based on an extension of the single index model of coincident indicators by Stock and Watson (1991). They conclude that both the world and the country specific factors captures a significant amount of the fluctuations. Kose et al. (2003) reaches similar conclusions, using a larger data set on 60 countries and using a Bayesian dynamic factor model. They conclude that real output growth depends on an international factor, a regional

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factor, plus an idiosyncratic one. The overall finding is again that the world factor explains a substantial fraction of economic fluctuations. In a recent paper, Kose et al. (2008) find however that the relative importance of the common factor has been declining over time and that the cycle of emerging economies has become decoupled from that of industrialized countries. Hess and Shin (1997, 1998) propose analysing the "intra-national" business cycle (i.e. the co-movements within a country) in order to gain understanding of the transmission mechanism of shocks that enables to abstract from the trade frictions that affect international economics. They use disaggregated U.S. State level data on productivity growth for several industries and assess, by a descriptive decomposition technique, the role of the common intra-national cycle, that of the industry specific and the state-specific cycles. They conclude that the role of the state specific cycle is much reduced and sector specific shocks are more important in a common currency area. Lumsdaine and Prasad (2003) assess the relative importance of country specific versus common shocks, using industrial production growth for a set of 17 countries. They estimate the common component of international fluctuations by the aggregation with time-varying weights (derived from the reciprocal of the conditional variance of the series, estimated by fitting a univariate GARCH model), which aims at downweighting the idiosyncratic variation, of the industrial production growth rates. In the present paper we focus on the business cycle of the European Monetary Union (EMU) and the cycles of 12 countries of the EMU. First, we aim to measure the cycle by using multivariate series and to extract the turning points of the country-specific business cycles. Secondly, we investigate the similarities between the EMU cycle at an aggregated level and the cycles of the 12 countries considered in our analysis.

Another aim of the paper is to verify the sources business cycle co-movements, i.e. on the channels through which business cycle fluctuations are transmitted across countries of the international economic system. We will focus on the following sources of transmission: interest rates (financial sector) and the oil prices (world shocks). In this respect, the literature has focused on the determinants on two main sources: trade and financial integration. Theoretically, there is no consensus in the literature on the role of trade in the international transmission of shocks. As argued by Frankel and Rose (1998), on the one hand trade has a positive direct impact on business cycle synchronisation, whilst on the other hand it could have an indirect negative effect through specialisation. Greater specialisation would lead to lower concordance, as countries may be more prone to sector-specific and idiosyncratic (or asymmetric) shocks (Bayoumi and Eichengreen (1993)). As a consequence, the direction of the link between trade openness and business cycle concordance is largely regarded as an empirical issue. Imbs (2004) estimates a simultaneous equations system to explain the observed cross-correlation of, say output growth, using explanatory variables that measure trade openness, financial integration and the degree of specialization. He concludes that trade has a strong effect on business cycle synchronization, but a sizable portion of this effect is found to actually work through intra-industry interlinkages. Financial integration also has a prevailing direct positive effect on synchronization. Canova and Marrinan (1998) address a different question, as to whether the international business

cycles originate from common shocks or from a common propagation mechanism. Monfort et al. (2003) aim at disentangling common shocks from spill-over effects. To this end, they estimate a Bayesian dynamic factor model for the G7 real output growth, featuring a global common factor and two area specific (North-American and Continental European) common factors, which, being modelled as a VAR process, are interdependent. They find empirical support for the presence of spill-over effects running from North-America to Continental Europe, but not vice versa.

This paper also contributes to the literature on heterogeneity in cross-country panel data models. Panel datasets are appealing because they combine the information coming from the cross-section and the time-series dimension of the data. In the context of the cross-country panel data models, the more recent approaches have focused on two issues: the estimation of international cycles and the nature of the co-movements using relatively large dimensional datasets and the introduction of country and time heterogeneity in multi country vector autoregressive models. The first issue has been considered by Hallin and Liska (2008), Pesaran et al. (2004), and Dees et al. (2007). The second by Canova and Ciccarelli (2006). Hallin and Liska (2008) extend the generalized dynamic factor model by Forni et al. (2000, 2001) to panel of time series with block structure, where the blocks are represented by countries. They show that the extension provides the means for the analysis of the interblock relationships, allowing the identification of strongly common factors, which are common to all the blocks (e.g. the international common factors), the strongly idiosyncratic factors, which are idiosyncratic for all blocks, and the weakly common/weakly idiosyncratic factors, that are common to at least one block, but idiosyncratic to at least another. Multi-country VAR models provide a tool for examining the propagation of shocks across countries. Canova and Ciccarelli (2006) consider Bayesian inference for multi-country VAR models with time varying parameters, lagged interdependencies and country specific effects. They avoid the curse of dimensionality by a factor parameterization of the time varying VAR coefficients in terms of a number of random effects that are linear in the number of countries and series. The random coefficients are in turn driven by a common component, a country specific component, a variable specific component and a idiosyncratic component. The factor loadings assumed to evolve according to a stationary vector first order autoregression, whereas the idiosyncratic component is assumed to be serially uncorrelated. The disturbances driving the evolution of the factors are also allowed to be heteroscedastic. The paper proposes a Monte Carlo Markov Chain sampling scheme to estimate the posterior distribution of the coefficients and to carry out impulse response analysis. Canova and Ciccarelli (2006) analyze the transmission of shocks in the G7 countries focusing on four macroeconomic variables: real growth, inflation, employment growth and rent inflation; oil prices are considered as exogenous. In this paper, we build on Canova and Ciccarelli (2006) and extend their panel VAR model in order to model asymmetry and the turning points in the business cycles of different countries. Our paper is also strictly related to Kaufmann (2010), where a panel of univariate Markov-switching (MS) regression models is considered. The early contributions in the business cycle literature

consider nonlinear models such as the MS models (see for example Goldfeld and Quandt (1973) and Hamilton (1989)) and the threshold autoregressive models (see Tong (1983) and Potter (1995)), both of which are able to capture the asymmetry and the turning points in business cycle dynamics. In this paper we focus on the class of MS models. We take the models of Hamilton (1989) and Krolzig (2000) as points of departure and consider Markov-switching dynamics for the VAR coefficients and covariance matrices.

The remainder of this paper is organized as follows. Section 1 presents the Bayesian panel MS-VAR model that has been used for the analysis. Section 2 discusses the prior choice and the Bayesian inference framework. Section 3 presents the empirical evidence on cross-country asymmetries in the business cycle and the comparison with the EMU and US cycle. The same session presents the asymmetries in the shocks transmission mechanism. Finally, Section 4 concludes.

2   A Panel Markov-switching VAR model

Let $\mathbf{y}_{it} \in \mathbb{R}^K$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$, be a sequence of K-dimensional vectors of observations. N is the number of units (countries) and T the number of time observations. We introduce a general specification of the panel Markov-switching VAR (PMS-VAR) model

$$\mathbf{y}_{it} = \mathbf{a}_i(s_{it}) + \sum_{j=1}^{N} \sum_{l=1}^{p} A_{ijl}(s_{it})\mathbf{y}_{jt-l} + D_i(s_{it})\mathbf{z}_t + \boldsymbol{\varepsilon}_{it}, \tag{1}$$

$i = 1, \ldots, N$, with $\boldsymbol{\varepsilon}_{it} \sim \mathcal{N}_K(\mathbf{0}, \Sigma_i(s_{it}))$ and $\mathbf{z}_t \in \mathbb{R}^G$ a vector of variables, common to all units.

The $\{s_{it}\}_t$ are unit-specific and independent M-states Markov-chain processes with values in $\{1, \ldots, M\}$ and transition probability $\mathbb{P}(s_{it} = k | s_{it-1} = j) = p_{i,kj}, j \in \{1, \ldots, M\}$. We assume the chains are stationary and irreducible. As regards to the choice of the number of regimes, we notice that for more recent data one needs an adequate business cycle model with more than two regimes (see also Clements and Krolzig (1998)) and a time-varying error variance. For example, Kim and Murray (2002) and Kim and Piger (2000) propose a three-regime (recession, high-growth, and normal-growth) MS model while Krolzig (2000) suggests the use of a model with regime-dependent volatility for the US GDP. In our paper we consider data on EMU industrial production, for a period of time including the 2009 recession and find that four regimes (high-recession, contraction, normal-growth, and high-growth) are necessary to capture some important features of the US and EMU cycle in the strong-recession phases.

The generality of the propose statistical model comes from the fact that the coefficients vary both across units and across time. Moreover the interdependencies between units are allowed whenever $A_{ijl}(s_{it}) \neq 0$ for $i \neq j$.

In order to define the parameter shifts more clearly and to simplify the exposition of

the inference procedure we introduce the indicator variable $\xi_{ikt} = \delta_k(s_{it})$, where

$$\delta_k(s_{it}) = \begin{cases} 1 & \text{if } s_{it} = k \\ 0 & \text{otherwise} \end{cases}$$

for $k = 1, \ldots, M$, $i = 1, \ldots, N$, and $t = 1, \ldots, T$ and the vector of indicators $\boldsymbol{\xi}_{it} = (\xi_{i1t}, \ldots, \xi_{iMt})'$, which collects the information about the realizations of the i-th unit-specific Markov chain over the sample period. The indicators allow us to write the parameter shifts as

$$\mathbf{a}_i(s_{it}) = \sum_{k=1}^{M} \mathbf{a}_{i,k}\xi_{ikt}, \quad A_{ijl}(s_{it}) = \sum_{k=1}^{M} A_{ijl,k}\xi_{ikt}$$

$$D_i(s_{it}) = \sum_{k=1}^{M} D_{i,k}\xi_{ikt}, \quad \Sigma_i(s_{it}) = \sum_{k=1}^{M} \Sigma_{i,k}\xi_{ikt}.$$

In our applications we will assume the following restrictions hold: $\mathbb{E}(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}_{jt}') = O_{K \times K}$ with $O_{n \times m}$ the $(n \times m)$-dimensional null matrix, and there are no interdependencies among the same variable across units, that is $A_{ijl,k} = A_{il,k}\delta_i(j) + O_{K \times K}(1 - \delta_i(j))$. Clements and Krolzig (1998) found in an empirical study that most forecast errors are due to the constant terms in the prediction models. They suggest considering, for example, MS models with regime-dependent volatility. In this paper, we follow Krolzig (2000) and Anas et al. (2008) and assume that both the unit-specific intercepts, $\mathbf{a}_i(s_{it})$, and volatilities, $\Sigma_i(s_{it})$, are driven by the regime-switching variables $\{s_{it}\}_t$ and assume constant autoregressive coefficients $A_{il,k} = A_{il}$, $\forall k$. In the same spirit we assume that the coefficients of the common variables do not change over time, that is $D_{i,k} = D_i$, $\forall k$.

Let $\bar{\mathbf{w}}_{it}' = (1, \ldots, \mathbf{y}_{it-1}', \cdots, \mathbf{y}_{it-p}', \mathbf{z}_t')$, $t = 1, \ldots, T$ be the sequence of $(1 + Kp + G)$-dimensional column vectors of regressors for the PMS-VAR model, that includes the constant term, the lagged dependent variables, and the set of common variables. Moreover define the regressors, $W_{it} = \bar{\mathbf{w}}_{it}' \otimes I_K$, and coefficients, $A_{i,k} = (\mathbf{a}_{i,k}, A_{i1,k}, \cdots, A_{ip,k}, D_i)$, matrices of dimension $(K(1 + Kp + G) \times K)$ and $(K \times K(1 + Kp + G))$ respectively. By using the allocation variables $\boldsymbol{\xi}_{it}$ and the unit independence assumptions, given above, the PMS-VAR model can be rewritten as

$$\mathbf{y}_{it} = A_{i,1}W_{it}\xi_{i1t} + \ldots + A_{i,M}W_{it}\xi_{iMt} + \boldsymbol{\varepsilon}_{it}, \quad \boldsymbol{\varepsilon}_{it} \sim \mathcal{N}_K(0, \Sigma_{it}) \tag{2}$$

or in a more compact form as $\mathbf{y}_{it} = (\boldsymbol{\xi}_{it} \otimes W_{it})\text{vec}(B_i) + \boldsymbol{\varepsilon}_{it}$ where $B_i = (\text{vec}(A_{i,1}), \text{vec}(A_{i,2}), \cdots, \text{vec}(A_{i,M}))$, $\Sigma_{it} = \Sigma_i(\boldsymbol{\xi}_{it} \otimes I_K)$ and $\Sigma_i = (\Sigma_{i1}, \cdots, \Sigma_{iM})$. For reason of convenience we consider the partition of the set of regressors $\bar{\mathbf{w}}_{it}$ into $M + 1$ subsets $\bar{\mathbf{x}}_{i0t}$ and $\bar{\mathbf{x}}_{imt}$, $m = 1, \ldots, M$, that are a K_0-dimensional vector of regressors with regime-invariant coefficients and M vectors of K_m regime-specific regressors with regime-dependent coefficients. Under this assumption the previous model writes as

$$\mathbf{y}_{it} = X_{i0t}\boldsymbol{\gamma}_{i0} + \xi_{i1t}X_{i1t}\boldsymbol{\gamma}_{i1} + \ldots + \xi_{iMt}X_{iMt}\boldsymbol{\gamma}_{iM} + \boldsymbol{\varepsilon}_{it} \tag{3}$$

where $X_{i0t} = (\bar{\mathbf{x}}'_{i0t} \otimes I_K)$ and $X_{imt} = (\bar{\mathbf{x}}'_{imt} \otimes I_K)$.

3 Bayesian Inference

3.1 Independent Priors

We assume a conjugate priors for the coefficients and the variance of the panel MS-VAR. For the coefficients $\boldsymbol{\gamma}_{i0}$ and $\boldsymbol{\gamma}_{im}$ we consider independent normals priors

$$\boldsymbol{\gamma}_{i0} \quad \sim \quad \mathcal{N}_{K_0}(\underline{\boldsymbol{\gamma}}_{i0}, \underline{\Sigma}_{i0}) \tag{4}$$

$$\boldsymbol{\gamma}_{im} \quad \sim \quad \mathcal{N}_{K_m}(\underline{\boldsymbol{\gamma}}_{im}, \underline{\Sigma}_{im}), \quad m = 1, \ldots, M \tag{5}$$

$i = 1, \ldots, N$. We assume independence across units, that is: $\mathbb{C}ov(\boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_{j0})) = 0$ and $\mathbb{C}ov(\boldsymbol{\gamma}_{im}, \boldsymbol{\gamma}_{jm})) = O_{K_m \times K_m}$, for $i \neq j$. For the inverse covariance matrix Σ_{im}^{-1} we assume the Wishart priors

$$\Sigma_{im}^{-1} \quad \sim \quad \mathcal{W}_K(\underline{\nu}_{im}/2, \underline{\Upsilon}_{im}/2), \quad m = 1, \ldots, M \tag{6}$$

with possibly regime-specific degrees of freedom $\underline{\nu}_{im}$ and precision $\underline{\Upsilon}_{im}$ parameters. We assume $\mathbb{C}ov(\Sigma_{im}^{-1}, \Sigma_{im}^{-1}) = O_{K_m^2 \times K_m^2}$.

When using Markov-switching processes, one should deal with the identification issue associated to the label switching problem. See for example Celeux (1998) and Frühwirth-Schnatter (2001) for a discussion on the effects of the label switching and the unidentification on the results of a MCMC based Bayesian inference. In the literature, different routes have been proposed for dealing with the label switching (see Frühwirth-Schnatter (2006) for a review). One of the most efficient approach is the permutation sampler (see Frühwirth-Schnatter (2001)), which can be applied under the assumption of exchangeability of the posterior distribution. This assumption satisfied when assuming symmetric prior on the transition probabilities of the switching process. As an alternative one could impose some identification constrains on the parameters. This practice is largely diffused in macroeconomics and is related to the natural interpretation of the different regimes as the different phases (e.g. recession and expansion) of the business cycle. In this work we follow this approach and include the constrains

$$\gamma_{ij1} < \gamma_{ij2} < \ldots < \gamma_{ijM}$$

$j = 1, \ldots, K$ and $i = 1, \ldots, N$, that corresponds to a total ordering, across the different regimes, of the constant terms in the equations of the system.

For the rows $\mathbf{p}_{i,j.}$, $j = 1, \ldots, M$, of the transition probability matrix we assume the independent Dirichlet distributions

$$\mathbf{p}_{i,j.} \sim \mathcal{D}(d_{i,1}, \ldots, d_{i,M}) \tag{7}$$

with $d_{i,j} = d_i$.

3.2 Hierarchical Prior

As an alternative to the independent prior assumption, a hierarchical priors could be used as in Canova and Ciccarelli (2006). This prior specification strategy allows to model dependence between the cross-sectional units through common latent variables. We will not consider hierarchical priors in our applications and briefly describe here a possible specification for further extensions of our work

$$
\begin{align}
\boldsymbol{\gamma}_{i0} &\sim \mathcal{N}_{K_0}(R_i\bar{\boldsymbol{\gamma}}_0, \underline{\Sigma}_{i0}) \tag{8}\\
\bar{\boldsymbol{\gamma}}_0 &\sim \mathcal{N}_{K_0}(\underline{\boldsymbol{\gamma}}_0, \underline{\Sigma}_0) \tag{9}\\
\boldsymbol{\gamma}_{im} &\sim \mathcal{N}_{K_m}(\bar{\boldsymbol{\gamma}}_m, \underline{\Sigma}_{im}), \quad m = 1,\ldots,M \tag{10}\\
\bar{\boldsymbol{\gamma}}_m &\sim \mathcal{N}_{K_m}(\underline{\boldsymbol{\gamma}}_m, \underline{\Sigma}_m), \quad m = 1,\ldots,M \tag{11}
\end{align}
$$

$i = 1,\ldots,N$, where $R_i = I_K$. We assume independence across units, that is: $\mathbb{C}ov(\boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_{j0}|\bar{\boldsymbol{\gamma}}_0) = O_{K_m \times K_m}$ and $\mathbb{C}ov(\boldsymbol{\gamma}_{im}, \boldsymbol{\gamma}_{jm}|\bar{\boldsymbol{\gamma}}_m) = 0$, for $i \neq j$. For the inverse covariance matrix Σ_{im}^{-1} we assume the Wishart priors

$$
\begin{align}
\Sigma_{im}^{-1} &\sim \mathcal{W}_K(\underline{\nu}_{im}/2, \bar{\Upsilon}_{im}/2), \quad m = 1,\ldots,M \tag{12}\\
\bar{\Upsilon}_m^{-1} &\sim \mathcal{W}_K(\underline{\nu}_m/2, \underline{\Upsilon}_m/2), \quad m = 1,\ldots,M \tag{13}
\end{align}
$$

$i = 1,\ldots,N$, that allow us to maintain the assumption of regime-specific degrees of freedom $\underline{\nu}_{im}$ and precision $\underline{\Upsilon}_{im}$ parameters. We assume $\mathbb{C}ov(\Sigma_{im}^{-1}, \Sigma_{im}^{-1}|\bar{\Upsilon}_m^{-1}) = O_{K_m^2 \times K_m^2}$.

Modeling dependence between the chains is a difficult issues to deal with. The hierarchical prior specification allow us to introduce dependence between the unit-specific Markov-chains. In a hierarchical prior setting there are many ways to introduce dependence. With the above given specification of the coefficients $\boldsymbol{\gamma}_{im}$ it is possible to have dependence between the different regimes. Another way to introduce dependence is through a hierarchical prior for the transition matrices. In particular for the i-th unit, the rows $\mathbf{p}_{i,j\cdot}$, $j = 1,\ldots,M$, of the transition probability matrix we assume

$$
\mathbf{p}_{i,j\cdot} \sim \mathcal{D}(d_{i,1}, \ldots, d_{i,M}) \tag{14}
$$

with $d_{i,j} = d$, that are conditionally independent and symmetric Dirichlet distributions. We assume $d \sim \mathcal{B}e(1/2, 1/2)$.

3.3 Gibbs sampler

We extend the Gibbs sampler of Krolzig (1997) and Frühwirth-Schnatter (2006) to our PMS-VAR model with the informative priors given in the previous sections. Under both the independent and hierarchical prior settings the full conditional posterior distributions

of the equation-specific blocks of parameters are independent. Thus the Gibbs sampler can be iterated over different blocks of parameters avoiding the computational difficulties associated with the inversions of large covariance matrices. We give the full conditional distributions of the parameters in Eq. 2. We apply a further blocking step. We follow the Markov-switching regression framework in Frühwirth-Schnatter (2006) and separate the unit-specific parameters into two different blocks: the regime-independent parameters and the regime-specific parameters.

The likelihood function associated to the PMS-VAR model is

$$p(\mathbf{y}|\Xi, \boldsymbol{\gamma}, \Sigma) = (2\pi)^{-\frac{TKN}{2}} \prod_{t=1}^{T} |\Sigma_t|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T} \mathbf{u}_t' \Sigma_t^{-1} \mathbf{u}_t\right\} \tag{15}$$

where $\mathbf{y}' = (\mathbf{y}_{11}', \ldots, \mathbf{y}_{N1}', \ldots, \mathbf{y}_{1T}', \ldots, \mathbf{y}_{NT}')$, $\Xi = (\boldsymbol{\xi}_{11}, \ldots, \boldsymbol{\xi}_{N1}, \ldots, \boldsymbol{\xi}_{1T}, \ldots, \boldsymbol{\xi}_{NT})$ and $\mathbf{u}_t = \mathbf{y}_t - ((1, \boldsymbol{\xi}_{1t}', \ldots, \boldsymbol{\xi}_{Nt}') \otimes I_{NK}) X_t \boldsymbol{\gamma}$. Under the independence assumption, the likelihood factorises as

$$\prod_{i=1}^{N} p(\mathbf{y}_i|\Xi_i, \boldsymbol{\gamma}_i, \Sigma_i) = \prod_{i=1}^{N} (2\pi)^{-\frac{TK}{2}} \prod_{t=1}^{T} |\Sigma_{it}|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\sum_{t=1}^{T} \mathbf{u}_{it}' \Sigma_{it}^{-1} \mathbf{u}_{it}\right\} \tag{16}$$

where $\mathbf{y}_i' = (\mathbf{y}_{i1}', \ldots, \mathbf{y}_{iT}')$, $\Xi_i = (\boldsymbol{\xi}_{i1}, \ldots, \boldsymbol{\xi}_{iT})$, $\boldsymbol{\gamma}_i' = (\boldsymbol{\gamma}_{i1}', \ldots, \boldsymbol{\gamma}_{iM}')$, $\mathbf{u}_{it} = \mathbf{y}_{it} - ((1, \boldsymbol{\xi}_{it}') \otimes I_K) X_{it} \boldsymbol{\gamma}_i$ and

$$X_{it} = \begin{pmatrix} X_{i0t} & X_{i1t} & & \mathbf{0} \\ \vdots & & \ddots & \\ X_{i0t} & \mathbf{0} & & X_{iMt} \end{pmatrix}$$

Let us introduce the auxiliary variables $\mathbf{y}_{i0t} = \mathbf{y}_{it} - \xi_{i1t} X_{i1t} \boldsymbol{\gamma}_{i1} + \ldots + \xi_{iMt} X_{iMt} \boldsymbol{\gamma}_{iM}$ and the notation $\boldsymbol{\gamma}_{i(-m)} = (\boldsymbol{\gamma}_{i1}, \ldots, \boldsymbol{\gamma}_{im-1}, \boldsymbol{\gamma}_{im+1}, \ldots, \boldsymbol{\gamma}_{iM})$ and $\Sigma_{i(-m)} = (\Sigma_{i1}, \ldots, \Sigma_{im-1}, \Sigma_{im+1}, \ldots, \Sigma_{iM})$.

Then the full conditional distribution of the regime-independent parameter $\boldsymbol{\gamma}_{i0}$ is a normal with density function

$$f(\boldsymbol{\gamma}_{i0}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_i, \Sigma_i) \propto \tag{17}$$

$$\propto \exp\left\{-\frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{y}_{i0t} - \boldsymbol{\gamma}_{i0})' \Sigma_{it}^{-1} (\mathbf{y}_{i0t} - \boldsymbol{\gamma}_{i0}) - \frac{1}{2}(\boldsymbol{\gamma}_{i0} - \underline{\boldsymbol{\gamma}}_{i0})' \Sigma_{i0}^{-1} (\boldsymbol{\gamma}_{i0} - \underline{\boldsymbol{\gamma}}_{i0})\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\boldsymbol{\gamma}_{i0}' \left(\sum_{t=1}^{T} X_{i0t}' \Sigma_{it}^{-1} X_{i0t} + \Sigma_{i0}^{-1}\right) \boldsymbol{\gamma}_{i0} + \boldsymbol{\gamma}_{i0} \left(\sum_{t=1}^{T} X_{i0t}' \Sigma_{it}^{-1} \mathbf{y}_{i0t} + \underline{\Sigma}_{i0}^{-1} \underline{\boldsymbol{\gamma}}_{i0}\right)\right\}$$

$$\propto \mathcal{N}_{K_0}(\bar{\boldsymbol{\gamma}}_{i0}, \bar{\Sigma}_{i0})$$

where $\bar{\boldsymbol{\gamma}}_{i0} = \bar{\Sigma}_{i0}^{-1}(\underline{\Sigma}_{i0}^{-1} \underline{\boldsymbol{\gamma}}_{i0} + \sum_{t=1}^{T} X_{i0t}' \Sigma_{it}^{-1} X_{i0t})$ and $\bar{\Sigma}_{i0}^{-1} = (\underline{\Sigma}_{i0}^{-1} + \sum_{t=1}^{T} X_{i0t}' \Sigma_{it}^{-1} X_{i0t})$.

The full conditional distributions of the regime-dependent parameters $\boldsymbol{\gamma}_{im}$, with $m =$

$1, \ldots, M$ are normal with density function

$$f(\boldsymbol{\gamma}_{im}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_{i(-m)}, \Sigma) \propto \tag{18}$$

$$\propto \exp\left\{-\frac{1}{2}\sum_{t\in\mathcal{T}_{im}}\mathbf{u}'_{it}\Sigma_t\mathbf{u}_{it} - \frac{1}{2}(\boldsymbol{\gamma}_{im}-\underline{\boldsymbol{\gamma}}_{im})'\underline{\Sigma}_{im}^{-1}(\boldsymbol{\gamma}_{im}-\underline{\boldsymbol{\gamma}}_{im})\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\boldsymbol{\gamma}'_i\left(\sum_{t\in\mathcal{T}_{im}}X'_{imt}\Sigma_{it}^{-1}X_{imt}+\underline{\Sigma}_{im}^{-1}\right)\boldsymbol{\gamma}_i + \boldsymbol{\gamma}'_i\left(\sum_{t\in\mathcal{T}_{im}}X'_{imt}\Sigma_{it}^{-1}\mathbf{y}_{imt}+\underline{\Sigma}_{im}^{-1}\underline{\boldsymbol{\gamma}}_{im}\right)\right\}$$

$$\propto \mathcal{N}_{K_m}(\bar{\boldsymbol{\gamma}}_{im}, \bar{\Sigma}_{im}) \tag{19}$$

with $\bar{\boldsymbol{\gamma}}_{im} = \bar{\Sigma}_{im}^{-1}(\underline{\Sigma}_{im}^{-1}\underline{\boldsymbol{\gamma}}_{im} + \sum_{\in\mathcal{T}_{im}}X'_{imt}\Sigma_{it}^{-1}X_{imt})$ and $\bar{\Sigma}_{im}^{-1} = (\underline{\Sigma}_{im}^{-1} + \sum_{\in\mathcal{T}_{im}}X'_{imt}\Sigma_{it}^{-1}X_{imt})$, where we defined $\mathcal{T}_{im} = \{t = 1, \ldots, T|\xi_{imt} = 1\}$ and $\mathbf{y}_{imt} = \mathbf{y}_{it} - X_{i0t}\boldsymbol{\gamma}_{i0}$.

The full conditional distributions of the regime-dependent inverse variance-covariance matrix Σ_{im}, with $m = 1, \ldots, M$ are Wishart distributions with density

$$f(\Sigma_{im}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_i, \Sigma_{i(-m)}) \propto \tag{20}$$

$$\propto \prod_{t=1}^{T}|\Sigma_{it}|^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\sum_{t\in\mathcal{T}_{im}}\mathbf{u}'_{it}\Sigma_{it}^{-1}\mathbf{u}_{it}\right\}|\Sigma_{im}^{-1}|^{\frac{\nu_{im}+K+1}{2}}\exp\left\{-\frac{1}{2}\text{tr}\left(\underline{\Upsilon}_{im}\Sigma_{im}^{-1}\right)\right\}$$

$$\propto |\Sigma_{im}^{-1}|^{\frac{\nu_{im}+T_{im}+K+1}{2}}\exp\left\{-\frac{1}{2}\text{tr}\left(\left(\underline{\Upsilon}_{im}+\sum_{t\in\mathcal{T}_{im}}\mathbf{u}_{imt}\mathbf{u}'_{imt}\right)\Sigma_{im}^{-1}\right)\right\}$$

$$\propto \mathcal{W}_K(\bar{\nu}_{im}/2, \bar{\Upsilon}_{im}/2) \tag{21}$$

where $T_{im} = \sum_{t=1}^{T}\mathbb{I}(\xi_{imt} = 1)$, $\mathbf{u}_{imt} = \mathbf{y}_{it} - X_{i0t}\boldsymbol{\gamma}_{i0} - X_{imt}\boldsymbol{\gamma}_{im}$, $\bar{\nu}_{im} = \underline{\nu}_i + T$ and $\bar{\Upsilon}_{im} = \underline{\Upsilon}_{im} + \sum_{t\in\mathcal{T}_{im}}\mathbf{u}_{imt}\mathbf{u}'_{imt}$. The full conditional distribution of the k-th row of the transition matrix is

$$f(\mathbf{p}_{i,k.}|\mathbf{y}_i, \Xi_i, \boldsymbol{\gamma}_{i0}, \boldsymbol{\gamma}_i) \propto \prod_{j=1}^{M}p_{i,kj}^{d_j}\prod_{t=1}^{T}\prod_{j=1}^{M}p_{i,kj}^{\xi_{ijt}\xi_{ikt}} \tag{22}$$

$$\propto \mathcal{D}(d_1 + N_{i,k1}, \ldots, d_M + N_{i,kM})$$

where

$$N_{i,kj} = \sum_{t=1}^{T}\mathbb{I}(s_{i,t} = j)\mathbb{I}(s_{i,t-1} = k)$$

counts the number of transitions of the i-th chain from the k-th to the j-th state.

In Krolzig (1997) the multi-move Gibbs sampler (see Carter and Kohn (1994) and Shephard (1994)) is presented for Markov-switching vector autoregressive models as an alternative to the single-move Gibbs sampler given, for example, in Albert and Chib (1993). The multi-move procedure, also known as forward-filtering backward sampling (FFBS) algorithm, is particularly useful in our context because the Gibbs sampler makes use of two relevant quantities, the filtering and the smoothing probabilities, that can be used for

turning point analysis.

The filtering probability at time t, $t = 1, \ldots, T$, is determined by iterating the prediction step

$$p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j | \mathbf{y}_{1:t-1}) = \sum_{i=1}^{m} p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j | \boldsymbol{\xi}_{t-1} = \boldsymbol{\iota}_i) p(\boldsymbol{\xi}_{t-1} = \boldsymbol{\iota}_i | \mathbf{y}_{1:t-1}) \qquad (23)$$

and the updating step

$$p(\boldsymbol{\xi}_t | \mathbf{y}_{1:t}) \propto p(\boldsymbol{\xi}_t | \mathbf{y}_{1:t-1}) p(y_t | \mathbf{y}_{t-1-p:t-1}, \boldsymbol{\xi}_t) \qquad (24)$$

where $p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j | \boldsymbol{\xi}_{t-1} = \boldsymbol{\iota}_i) = p_{ij}$, with $\boldsymbol{\iota}_m$ the m-th column of the identity matrix and $p(y_t | \mathbf{y}_{t-p-1:t-1}, \boldsymbol{\xi}_t)$ the conditional distribution of the variable y_t from a MSIH(m)-AR(p).

We shall notice that the prediction step can be used at time t to find the predictive density of $\boldsymbol{\xi}_{t+1}$

$$p(\boldsymbol{\xi}_{t+1} | \mathbf{y}_{1:t}) \propto P' \, p(\boldsymbol{\xi}_t | \mathbf{y}_{1:t}) \qquad (25)$$

and the one of y_{t+1}

$$p(y_{t+1} | \mathbf{y}_{1:t}) = \sum_{i=1}^{m} p(\boldsymbol{\xi}_{t+1} = \boldsymbol{\iota}_i | \mathbf{y}_{1:t}) p(y_{t+1} | \mathbf{y}_{t+1-p:t}, \boldsymbol{\xi}_{t+1}) \qquad (26)$$

which, for a Gaussian MS-AR process, is a discrete mixture of normal distributions.

The smoothing probabilities given by

$$p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j | \mathbf{y}_{1:T}) \propto \sum_{i=1}^{m} p(\boldsymbol{\xi}_t = \boldsymbol{\iota}_j | \boldsymbol{\xi}_{t+1} = \boldsymbol{\iota}_i, \mathbf{y}_{1:t}) p(\boldsymbol{\xi}_{t+1} = \boldsymbol{\iota}_i | \mathbf{y}_{1:T}) \qquad (27)$$

are evaluated recursively and backward in time for $t = T, T-1, \ldots, 1$. These quantities are the posterior probabilities of the observation y_t to be in one of the m regimes at time t, given all the information available from the full sample of data. The conditional distribution $p(\boldsymbol{\xi}_t | \boldsymbol{\xi}_{t+1}, \mathbf{y}_{1:t})$, that is the building block of the smoothing probability formula, is used in the FFBS algorithm to sample the allocation variables from their joint posterior distribution sequentially and backward in time for $t = T, T-1, \ldots, 1$. See Frühwirth-Schnatter (2006), ch. 11-13, for further details.

As discussed in previous sections, when using data-dependent priors the generation of the allocation variables should omit draws that yield to impropriety of the posterior. In our prior settings, the set of non-troublesome grouping is $\mathcal{S} = \mathcal{S}_\nu \cap \mathcal{S}_\sigma = \mathcal{S}_\sigma$. Thus, each time the set of allocation variables $\boldsymbol{\xi}_{1:T}$, does not assign at least two observations to each component of the dynamic mixture, the entire set $\boldsymbol{\xi}_{1:T}$, is rejected and a new set is drawn until a proper set is obtained.

The smoothing probabilities are usually employed also to detect the turning points. In this paper, we will not consider the cycle generated by the smoothing probabilities and instead applied a non-parametric approach (see the next section) to extract the turning points from the forecasting values of y_{t+h}.

3.4 Regime Probability Combination

Let $\Delta_{[0,1]^M}$ be the standard simplex and $\boldsymbol{\eta}_{i,t} \in \Delta_{[0,1]^M}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$, be a sequence M-dim vectors of smoothing (or predictive) probabilities for the M different regimes of the N unit-specific Markov-chains used in the PMS-VAR model. These probabilities reveal information on the dynamics of the endogenous variables both at the unit-specific and aggregated levels. We propose a method to summarize the information contents of the different units. We combine the smoothing (or predictive) probabilities and get a new probability vector sequence, $\boldsymbol{\eta}_t \in \Delta_{[0,1]^M}$, $t = 1, \ldots, T$. We define a general aggregation scheme as a map $\boldsymbol{\phi} : \Delta_{[0,1]^M}^N \mapsto \Delta_{[0,1]^M}$

$$\boldsymbol{\eta}_t = \boldsymbol{\phi}(\boldsymbol{\eta}_{1,t}, \ldots, \boldsymbol{\eta}_{N,t}) \tag{28}$$

such that $\boldsymbol{\eta}_t \in \Delta_{[0,1]^M}$, that is $\boldsymbol{\eta}_t$ can be interpreted as a probability.

We consider here two alternative aggregation schemes:

- **Equal weights**

  Let

  $$\hat{s}_{i,t} = \underset{k \in \{1, \ldots, M\}}{\arg \max} \{\eta_{i1,t}, \ldots, \eta_{iM,t}\}$$

  the MAP estimate of the unit-specific regime at time t. A simple aggregation method is

  $$\eta_{kt} = \frac{1}{N} \sum_{i=1}^{N} \delta_k(\hat{s}_{i,t}) \tag{29}$$

  $k = 1, \ldots, M$, where we assigned equal weights to the unit-specific regime probabilies. When $k = 1$ we get a measure of the proportion of countries which are in a "'strong recession"' regime.

- **Unit-specific weights**

  Let $\hat{s}_{i,t}$ as above, then we define the second combination scheme

  $$\eta_{kt} = \sum_{i=1}^{N} \omega_{it} \delta_k(\hat{s}_{i,t}) \tag{30}$$

  where, in order to have a properly defined vector of probability, we assume $(\omega_{1t}, \ldots, \omega_{Nt})' \in \Delta_{[0,1]^N}$. The unit-specific weight ω_{it}, can be driven, for example, by the relative IPI growth rate or IPI size of the i-th unit in the sample with respect to the other units.
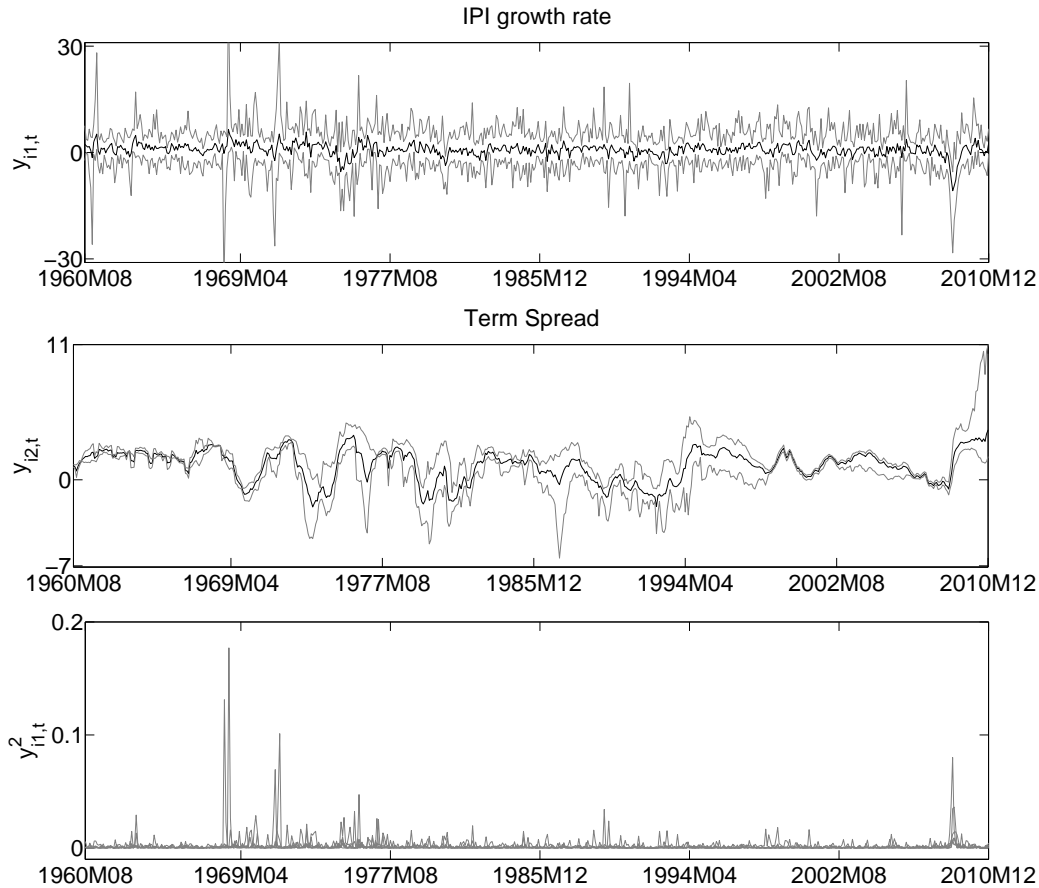
Figure 1: Top: log-change in percent (top chart) of the EMU area Industrial Production Indexes (IPI). Middle: term spread (TS), that is the difference between 3-month and 10-year interest rates. All variables are at a monthly frequency for the period: January 1960 to December 2010. Black lines: average value of the variable across countries. Gray lines: maximum and minimum values across countries. Bottom: square of the IPI log-change series.

4 Business Cycle Analysis

4.1 Data Description

As dependent variables in our PMS-VAR model we consider for $y_{i1,t}$ the Industrial Production Index (IPI) and for $y_{i2,t}$ the short term (3 months) and long term (10 years) interest rate differentials, for the EMU area. All data are from the Eurostat and OECD databases and are sampled at a monthly frequency, from January 1960 to December 2010. As our aim is to analyse the individual contribution of the EMU countries to the fluctuations of the EMU area business cycle, we do not consider the variables at the Euro zone level, but at a country level. More specifically we consider IPI and interest rates for 12 countries: Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. Data for the EMU countries are seasonally adjusted and working day adjusted. The data are available with different sample sizes for

Begin dates of the series			
Country	IPI	3m-IR	10y-IR
Austria	1960M01	1989M06	1990M01
Belgium	1960M01	1960M01	1960M01
Finland	1960M01	1987M01	1988M01
France	1960M01	1970M01	1960M01
Germany	1960M01	1960M01	1960M01
Greece	1962M01	1997M06	2001M01
Ireland	1975M07	1984M01	1970M12
Italy	1960M01	1978M10	1991M03
Luxembourg	1960M01	1999M01	1993M10
Netherlands	1960M01	1986M01	1960M01
Portugal	1960M01	1992M01	1993M07
Spain	1965M01	1977M01	1980M01

Table 1: Begin date for the series of the Industrial Production Index (IPI) and of the 3-months (3m-IR) and 10-years (10y-IR) interest rates in 12 countries of the EMU. The end date for all of the series is December, 2010.

the EMU countries (see Tab. 1). The problem of sample with different sizes has been handled in a Bayesian setting, through a suitable specification of the prior distribution (see Section 3). Moreover, since Phillips-Perron and Dickey-Fuller stationarity tests point out the non-stationarity of the IPI, we considered in our analysis the log-changes of the IPI index.

Another aim of the analysis is to capture the shock transmission mechanism from the financial sector to the real one. We consider, as a source of financial shocks, the spread between long and short interest rates. For the EMU countries, interest rate data are available with different sample sizes (see Tab. 1). As a source of global shocks for the EMU area we consider log-changes in the oil West Texas Index (WTI) of spot prices, that is available from the Bloomberg database, from January 1961.

We apply the proposed PMS-VAR model to IPI grow rate and term spread series (upper and mid charts in Fig. 1). The presence of time-varying volatility and volatility clustering (bottom chart in Fig. 1) suggests that the model should account for different regimes in the volatility level.

4.2 Parameter Estimates

The posterior distributions of the PMS-VAR model parameters are approximated through a kernel density estimator applied to a sample of 1,000 random draws from the posterior. In order to generate 1,000 i.i.d. samples from the posterior, we run the Gibbs sampler, given in Section 3, for 110,000 iterations, discard the first 10,000 draws to avoid dependence from the initial condition, and finally apply a thinning procedure with a factor of 100 samples, to reduce the dependence between consecutive Markov-chain draws. As regards to the number of iterations, we should say that the choice of the initial sample size and the convergence

detection of the Gibbs sampler remain open issues (see Robert and Casella (1999)). In our application we choose the sample size on the basis of both a graphical inspection of the MCMC progressive averages and the application of the convergence diagnostic (CD) statistics proposed in Geweke (1992). We let $n = 110,000$ be the MCMC sample size and $n_1 = 10,000$, and $n_2 = 30,000$ the sizes of two non-overlapping sub-samples. For a parameter θ of interest, we let

$$\hat{\theta}_1 = \frac{1}{n_1} \sum_{j=1}^{n_1} \theta^{(j)}, \quad \hat{\theta}_2 = \frac{1}{n_2} \sum_{j=n+1-n_2}^{n} \theta^{(j)}$$

be the MCMC sample means and $\hat{\sigma}_i^2$ their variances estimated with the non-parametric estimator

$$\frac{\hat{\sigma}_i^2}{n_i} = \hat{\Gamma}(0) + \frac{2n_i}{n_i - 1} \sum_{j=1}^{h_i} K(j/h_i)\hat{\Gamma}(j),$$

$$\hat{\Gamma}(j) = \frac{1}{n_i} \sum_{k=j+1}^{n_i} (\theta^{(k)} - \hat{\theta}_i)(\theta^{(k-j)} - \hat{\theta}_i)'$$

where we choose $K(x)$ to be the Parzen kernel (see Kim and Nelson (1999)) and $h_1 = 100$ and $h_2 = 500$ the bandwidths. Then the following statistics

$$CD = \frac{\hat{\theta}_1 - \hat{\theta}_2}{\sqrt{\hat{\sigma}_1^2/n_1 + \hat{\sigma}_2^2/n_2}} \tag{31}$$

converges in distribution to a standard normal (see Geweke (1992)), under the null hypothesis that the MCMC chain has converged.

Figures 2 and 3 show the approximated posterior distributions of the parameters $\gamma_{im} = (a_{i1,m}, a_{i2,m})'$, $(\sigma_{i\,11,m})$ and $(\sigma_{i\,22,m})$, $m = 1,\ldots,M$ and $i = 1,\ldots,N$, that represent the value of the unit- and variable-specific time-varying intercepts and volatilities of the PMS-VAR model. The posterior mean and the credibility region of the parameters $\gamma_{im} = (a_{i1,m}, a_{i2,m})'$ and $\Sigma_{im} = (\sigma_{i\,kj,m})_{k<j}$, are given in Tab. 2-4.

As regards to the intercept posterior ((see first column of Fig. 2)), there are at least two groups of countries. The first one is Belgium, France and Germany, with intercept parameters, $a_{i1,m}$, for the IPI growth rate, that do not differ to much across the regimes, $m = 1, 2, 3$, (see coloured lines within each chart in Fig. 2). From Tab. 2 the average intercept values are -0.17, -0.27 and 0.2 for the first, second and third regime, respectively. The rage of variation of the intercept parameters, $a_{i1,m}$, of the remaining group of countries, that are Austria, Finland, Greece, Ireland, Italy, Luxembourg, Netherland, Portugal and Spain, differ substantially across the regimes, in terms of location and shape. The average intercept values are -3.635, -0.57 and 3.365 in the first, second and third regime, respectively. Within the second group, Austria, Portugal and Spain have similar intercept posteriors, in terms of location and dispersion, across the first (strong recession) and the second regime
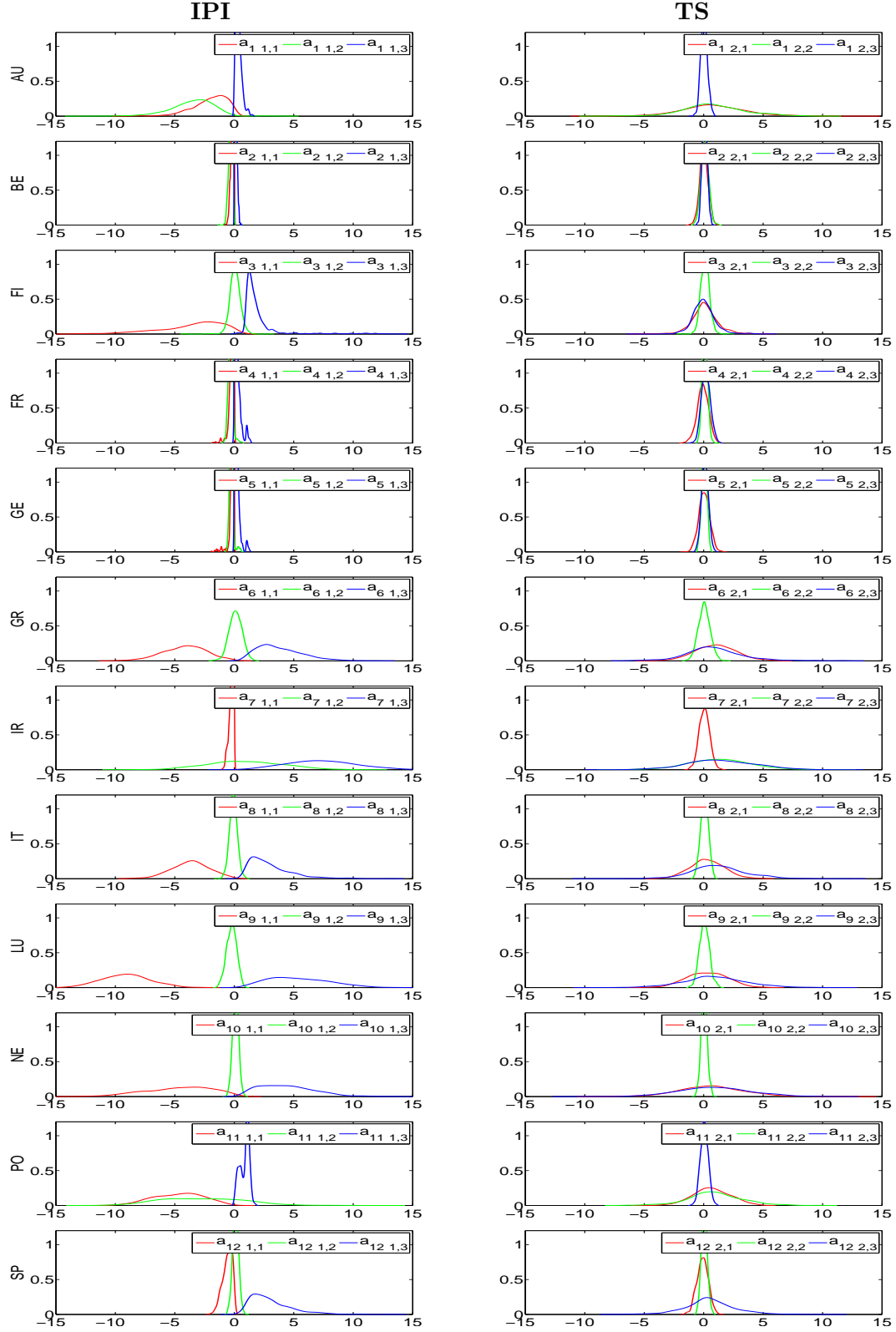
Figure 2: Posterior distribution of the Markov-switching intercepts, $\gamma_{im} = (a_{i1,m}, a_{i2,m})'$, $i = 1, \ldots, N$, $m = 1, \ldots, M$ for IPI growth rate (left column) and TS (right column).
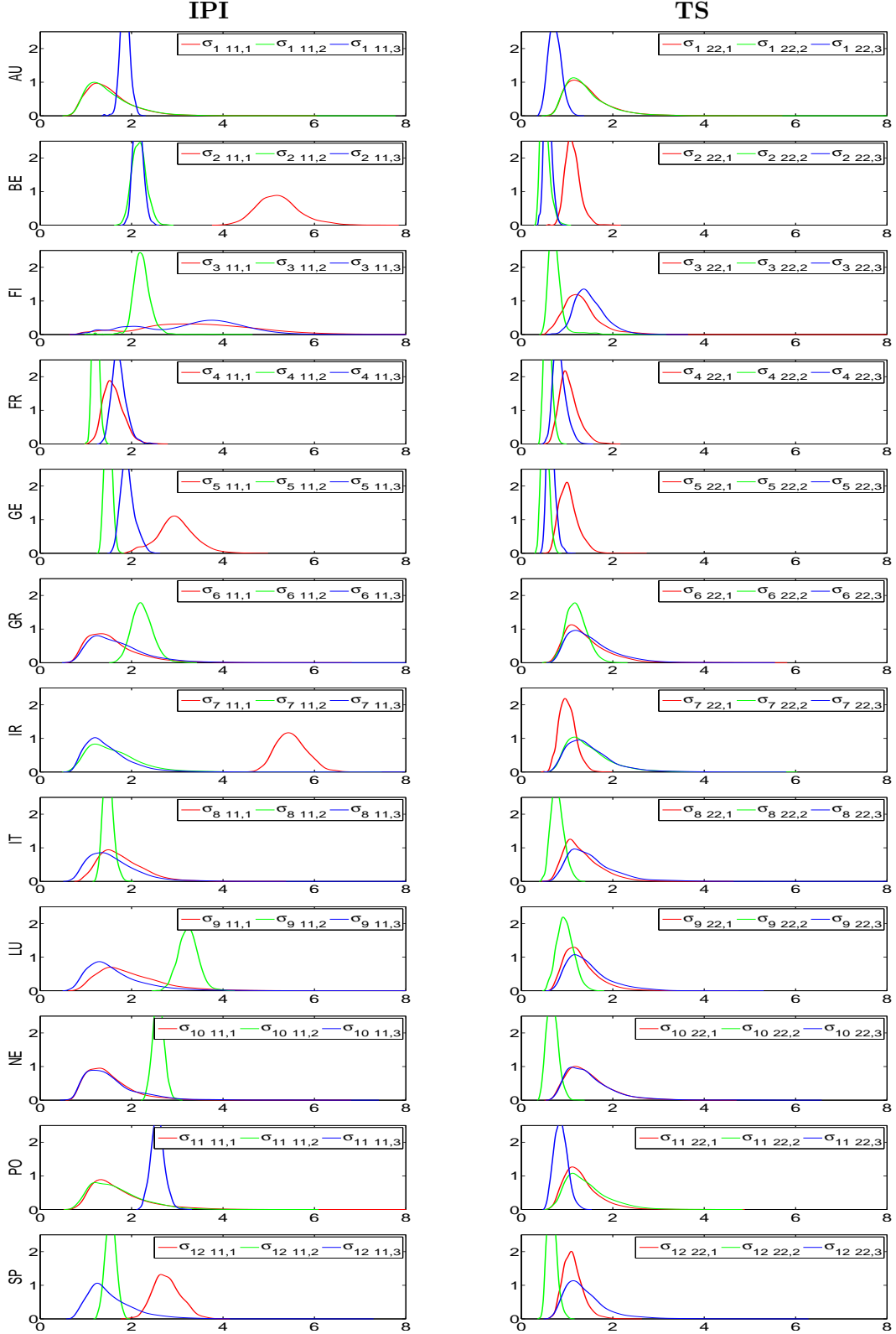
Figure 3: Posterior distribution of the square root of the diagonal elements, $\sigma_{ikj,m}$, $k, j = 1, \ldots, K$, with $k = j$, of the Markov-switching covariance matrices, Σ_{im}, $i = 1, \ldots, N$ and $m = 1, \ldots, M$, for IPI (left column) and TS (right column).

Country	i	Regime 1 $a_{i1,1}$	$a_{i2,1}$	$\sigma_{i\,11,1}$	$\sigma_{i\,22,1}$	$\sigma_{i\,12,1}$
AU	1	-2.0242	0.8453	1.526	1.4309	0.0283
		(-4.68,-.19)	(-3.61,5.64)	(.9,2.51)	(.87,2.37)	(-4.13,3.46)
BE	2	-0.1438	0.0127	5.213	1.1209	-0.5786
		(-.39,-.01)	(-.54,.58)	(4.52,6.08)	(.89,1.41)	(-1.82,.64)
FI	3	-3.5669	0.1015	3.5118	1.3203	-0.331
		(-8.93,-.24)	(-1.68,2.13)	(1.41,5.64)	(.78,2.11)	(-4.14,3.46)
FR	4	-0.2063	-0.0703	1.601	1.0404	-0.0537
		(-.53,-.01)	(-.89,.66)	(1.3,1.97)	(.74,1.43)	(-.71,.61)
GE	5	-0.1704	0.046	2.9743	1.0345	-0.253
		(-.41,-.01)	(-.7,.81)	(2.27,3.64)	(.75,1.42)	(-1.23,.67)
GR	6	-4.1517	1.013	1.5835	1.4095	-0.0286
		(-7.24,-1.24)	(-1.77,3.79)	(.92,2.65)	(.85,2.31)	(-4.05,4.12)
IR	7	-0.2211	0.0583	5.4996	1.008	-0.0973
		(-.62,-.02)	(-.64,.78)	(5.,6.08)	(.74,1.33)	(-1.08,.9)
IT	8	-3.6763	0.2934	1.7706	1.3322	-0.1696
		(-6.33,-1.1)	(-2.07,2.67)	(1.14,2.66)	(.83,2.1)	(-3.14,2.4)
LU	9	-9.1082	0.277	1.9724	1.2999	0.0593
		(-12.43,-5.56)	(-2.46,2.99)	(1.1,3.36)	(.84,2.02)	(-5.56,5.72)
NE	10	-4.6717	0.6634	1.5039	1.463	0.0532
		(-9.52,-.67)	(-3.75,5.21)	(.9,2.51)	(.87,2.37)	(-4.45,5.02)
PO	11	-4.7046	0.6718	1.7238	1.3152	-0.2942
		(-8.14,-1.42)	(-1.89,3.28)	(1.02,2.98)	(.83,2.07)	(-4.33,3.35)
SP	12	-0.5913	-0.0935	2.7753	1.1279	-0.2135
		(-1.32,-.05)	(-.87,.65)	(2.33,3.28)	(.82,1.52)	(-1.2,.8)

Table 2: Posterior mean and credible intervals (in parenthesis) for the parameters, $\gamma_{im} = (a_{i1,m}, a_{i2,m})'$ and $\Sigma_{im} = (\sigma_{ijk,m})_{j<k}$, $m = 1$ (first regime) and $i = 1, \ldots, N$, which are driven by the Markov-switching processes. The estimates are obtained with 1,000 draws, that are the result of 110,000 iterations of the Gibbs sampler, of a burn-in period of 10,000 draws and a thinning procedure with a thinning factor of 100 samples.

(moderate growth or recession). The posterior distribution of the unit- and variable-specific volatilities (see first column of Fig. 3 in the different regimes (different line within the same chart) are quite different across regimes. Belgium, Finland, Germany, Ireland and Spain exhibit a high volatility (red lines) associated with the first regime (recession) with respect to the volatility of the moderate recession/growth (green line) and expansion regimes. The posterior distribution of the volatilities of the first and second regime are quite similar, for Austria and Portugal, while for Belgium and Ireland, the volatilities in the second and third regime are similar. For Italy all of the three regimes exhibit similar volatility features.

4.3 Turning Points

The PMS-VAR model allows us to study the business cycles fluctuations of each country in the panel, to analyse the transmission of shocks across cycles and predict the turning points of the country-specific cycles. The red lines in Fig. 4 present the country-specific cycles in terms of a 3-regime Markov-chain. The regimes are: strong recession, $s_{i,t} = 1$, moderate

Country	i	Regime 2				
		$a_{i1,2}$	$a_{i2,2}$	$\sigma_{i\,11,2}$	$\sigma_{i\,22,2}$	$\sigma_{i\,12,2}$
AU	1	-3.5019	0.8714	1.5161	1.4219	-0.0547
		(-6.79,-1.04)	(-3.12,5.81)	(.92,2.49)	(.87,2.35)	(-4.14,3.3)
BE	2	-0.2779	0.0924	2.1663	0.5164	-0.0333
		(-.57,-.06)	(-.4,.61)	(1.93,2.43)	(.39,.71)	(-.34,.25)
FI	3	0.0649	0.1184	2.2306	0.7454	-0.0026
		(-.6,.77)	(-.45,.73)	(1.97,2.54)	(.55,1.01)	(-.35,.3)
FR	4	-0.2968	0.0977	1.2154	0.578	-0.0166
		(-.6,-.05)	(-.24,.46)	(1.11,1.33)	(.46,.72)	(-.15,.11)
GE	5	-0.2357	0.0503	1.4882	0.5187	0.0129
		(-.46,-.05)	(-.3,.38)	(1.37,1.62)	(.4,.65)	(-.14,.15)
GR	6	0.112	0.0451	2.2352	1.2036	-0.0777
		(-.73,.98)	(-.74,.85)	(1.88,2.63)	(.87,1.57)	(-.84,.72)
IR	7	0.9287	1.3359	1.617	1.4704	-0.0663
		(-4.19,6.17)	(-2.99,5.87)	(.91,2.69)	(.88,2.41)	(-5.51,4.91)
IT	8	-0.0771	0.0687	1.4874	0.7886	0.0347
		(-.62,.46)	(-.48,.59)	(1.32,1.68)	(.58,1.03)	(-.25,.31)
LU	9	-0.226	0.0602	3.2394	0.9625	0.0592
		(-.95,.46)	(-.61,.71)	(2.9,3.59)	(.68,1.27)	(-.56,.67)
NE	10	0.1114	0.0366	2.5878	0.6758	-0.0144
		(-.3,.52)	(-.38,.45)	(2.4,2.78)	(.5,.89)	(-.27,.25)
PO	11	-2.6537	0.9577	1.662	1.431	-0.1303
		(-7.89,2.81)	(-2.8,4.86)	(.96,2.71)	(.88,2.33)	(-4.7,4.56)
SP	12	0.1082	0.0542	1.5429	0.6671	0.0041
		(-.31,.51)	(-.34,.47)	(1.36,1.73)	(.52,.85)	(-.2,.21)

Table 3: Posterior mean and credible intervals (in parenthesis) for the parameters, $\gamma_{im} = (a_{i1,m}, a_{i2,m})'$ and $\Sigma_{im} = (\sigma_{i\,jk,m})_{j<k}$, $m = 2$ (second regime) and $i = 1, \ldots, N$, which are driven by the Markov-switching processes. The estimates are obtained with 1,000 draws, that are the result of 110,000 iterations of the Gibbs sampler, of a burn-in period of 10,000 draws and a thinning procedure with a thinning factor of 100 samples.

recession or moderate expansion, $s_{i,t} = 2$, and strong expansion, $s_{i,t} = 3$). The smoothed probabilities of the three regimes, $\tilde{p}_{i,t} = \mathbb{P}(\tilde{s}_{i,t} = 1|\mathbf{y}_{1:T})$, for $i = 1, \ldots, N$ are given in Figures from 6 to 8. We observe that the regimes are often highly persistent, excluding few cases at the end of 80's and beginning of 90's where few recessions were estimated with very short life, see e.g. Finland and Ireland. On average, regime 2 is the most probable as we could anticipate since its definition can fit both light recession and expansion periods. The 70's and beginning of 80's are the most volatile, with several periods of strong recessions, but also strong expansion. The great moderation and the great financial crisis in 2008-2009 are also evident. The exception is Ireland which is estimated to be in regime 1 from the end of 90's. The Irish economy had experienced substantial changes from the 90's switching from farms and light industries to services. Our model suggests that the Irish economy was underperforming conditional to the low term spread.

In order to have a measure of the contagion of the recession within the EMU area we apply the combination methods given in Eq. 29 and 30. Both measures in Figure 5 indicate that the great financial crisis was the period with longer and stronger recession

Country	i $a_{i1,3}$	$a_{i2,3}$	$\sigma_{i\,11,3}$	$\sigma_{i\,22,3}$	$\sigma_{i\,12,3}$
			Regime 3		
AU	1 0.3668	0.0344	1.8541	0.7307	-0.027
	(.04,.84)	(-.4,.51)	(1.7,2.02)	(.52,.97)	(-.29,.24)
BE	2 0.1099	0.0857	2.1417	0.5782	-0.0143
	(.01,.29)	(-.25,.42)	(1.98,2.32)	(.46,.72)	(-.18,.15)
FI	3 2.0781	0.0295	3.2925	1.4981	0.273
	(1.03,5.88)	(-1.37,1.68)	(1.4,4.8)	(1.05,2.09)	(-2.82,3.28)
FR	4 0.2657	0.1885	1.7246	0.8334	-0.04
	(.01,1.01)	(-.39,.83)	(1.5,2.)	(.65,1.08)	(-.42,.36)
GE	5 0.223	0.1074	1.9156	0.6455	0.04
	(.01,.64)	(-.35,.57)	(1.7,2.21)	(.52,.8)	(-.17,.26)
GR	6 3.7886	0.8778	1.6864	1.5113	-0.0768
	(1.32,7.34)	(-2.63,5.09)	(.95,2.83)	(.9,2.5)	(-4.35,4.01)
IR	7 7.3548	1.4607	1.4935	1.496	-0.1523
	(2.64,12.55)	(-3.31,6.23)	(.88,2.47)	(.89,2.45)	(-4.27,4.65)
IT	8 2.9927	1.2244	1.5941	1.5433	0.1307
	(1.1,6.68)	(-2.28,5.32)	(.93,2.61)	(.91,2.5)	(-4.36,4.66)
LU	9 5.4322	0.9447	1.6577	1.4504	-0.0798
	(1.77,10.01)	(-3.63,5.71)	(.91,2.99)	(.89,2.39)	(-5.3,4.91)
NE	10 4.3871	0.9305	1.5606	1.4971	0.2215
	(1.3,8.4)	(-3.86,6.11)	(.88,2.71)	(.89,2.53)	(-4.19,4.55)
PO	11 0.8666	0.0554	2.5702	0.8614	-0.0181
	(.17,1.37)	(-.46,.59)	(2.34,2.82)	(.64,1.11)	(-.42,.37)
SP	12 3.0198	0.508	1.5576	1.4046	0.1818
	(1.09,6.61)	(-2.89,4.2)	(.94,2.6)	(.86,2.37)	(-3.18,3.45)

Table 4: Posterior mean and credible intervals (in parenthesis) for the parameters, $\boldsymbol{\gamma}_{im} = (a_{i1,m}, a_{i2,m})'$ and $\Sigma_{im} = (\sigma_{ijk,m})_{j<k}$, $m = 3$ (third regime) and $i = 1, \ldots, N$, which are driven by the Markov-switching processes. The estimates are obtained with 1,000 draws, that are the result of 110,000 iterations of the Gibbs sampler, of a burn-in period of 10,000 draws and a thinning procedure with a thinning factor of 100 samples.

period. However, the equal weight averages shows that not all countries were in the deepest point at the same time, calling for an analysis which allow for the possibility of leading and lagging countries. The weighted average is close to 1 in several periods and it is 1 for several consecutive months during the first and second oil shock in the 70's, the end of 80's and the great financial crisis. The latter index is, however, highly volatile with some very short living false signals.

5 Conclusion

We propose a new Bayesian panel VAR model with unit-specific Markov-switching latent factors. We discuss the choice of the prior with particular attention to the case that some variable are missing. We apply the resulting panel MS-VAR model and the simulation-based Bayesian inference procedure to the analysis of the contributions of the EMU countries to the fluctuations of the EMU business cycle. We extract the turning points of the the unit-specific business cycle and propose an aggregation technique for the reconstruction of the
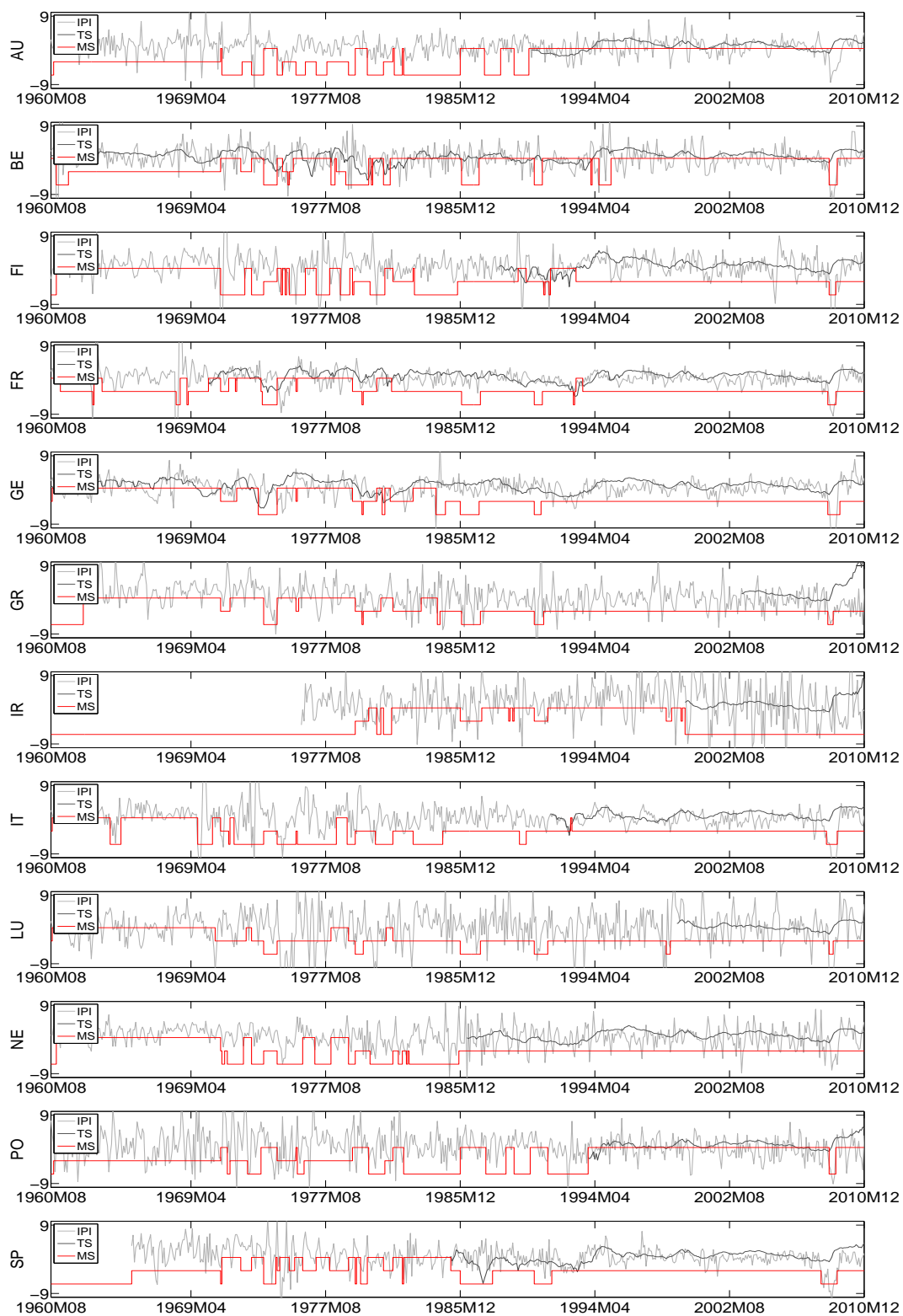
Figure 4: Country-specific endogenous variables: industrial production growth rate (IPI) and term structure (TS); and Markov-switching (MS) processes $s_{i,t}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$.
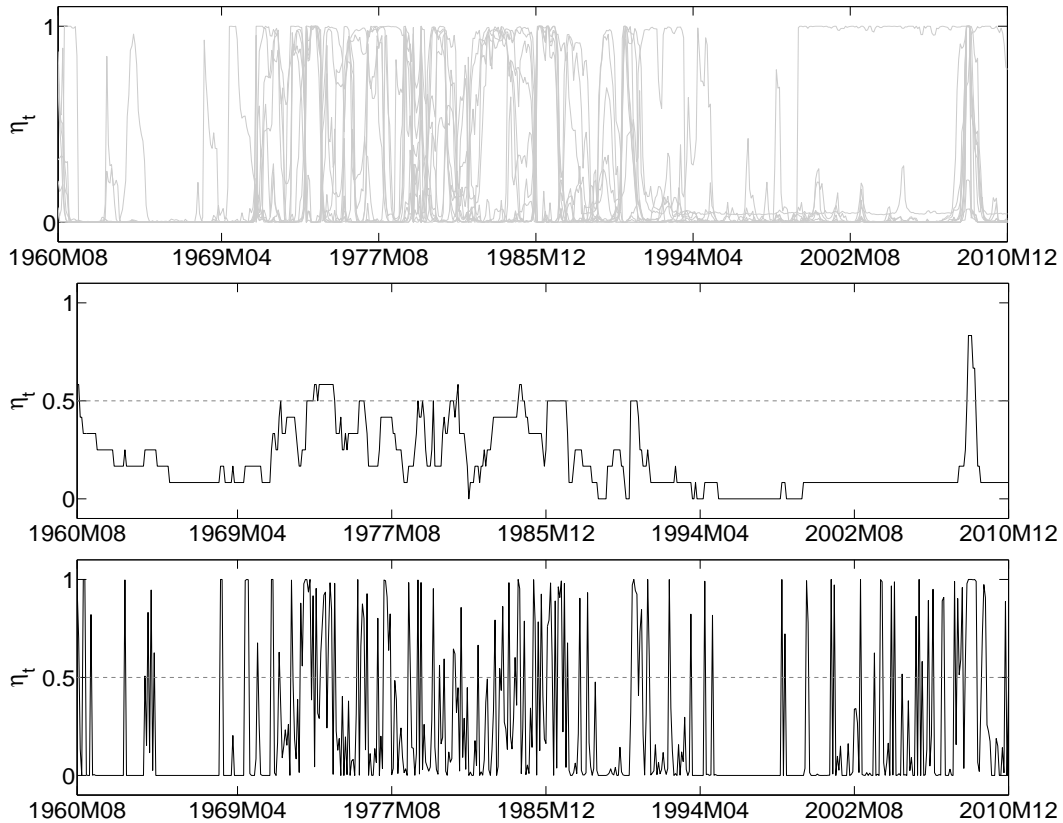
Figure 5: Smoothed probability (top) of being in the recession regime (regime 1) for the Markov-switching processes $s_{i,t}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$. Proportion (middle) and weighted proportion (bottom) of countries in a "'strong recession"' regime.

EMU turning points.

6 Appendix A

References

Albert, J. H. and Chib, S. (1993). Bayes inference via Gibbs sampling of autoregressive time series subject to Markov mean and variance shifts. *Journal of Business and Economic Statistics*, 11:1–15.

Anas, J., Billio, M., Ferrara, L., and Mazzi, G. L. (2008). A System for Dating and Detecting Turning Points in the Euro Area. *The Manchester School*, 76:549–577.

Bayoumi, T. and Eichengreen, B. (1993). Shocking Aspects of European Monetary Unification. In Giavazzi, F. and Torres, F., editors, *The Transition to Economic and Monetary Union in Europe*. Cambridge University Press.

Canova, F. and Ciccarelli, M. (2006). Estimating Multi-Country VAR Models. ECB working paper No 603. European Central Bank, Frankfurt.
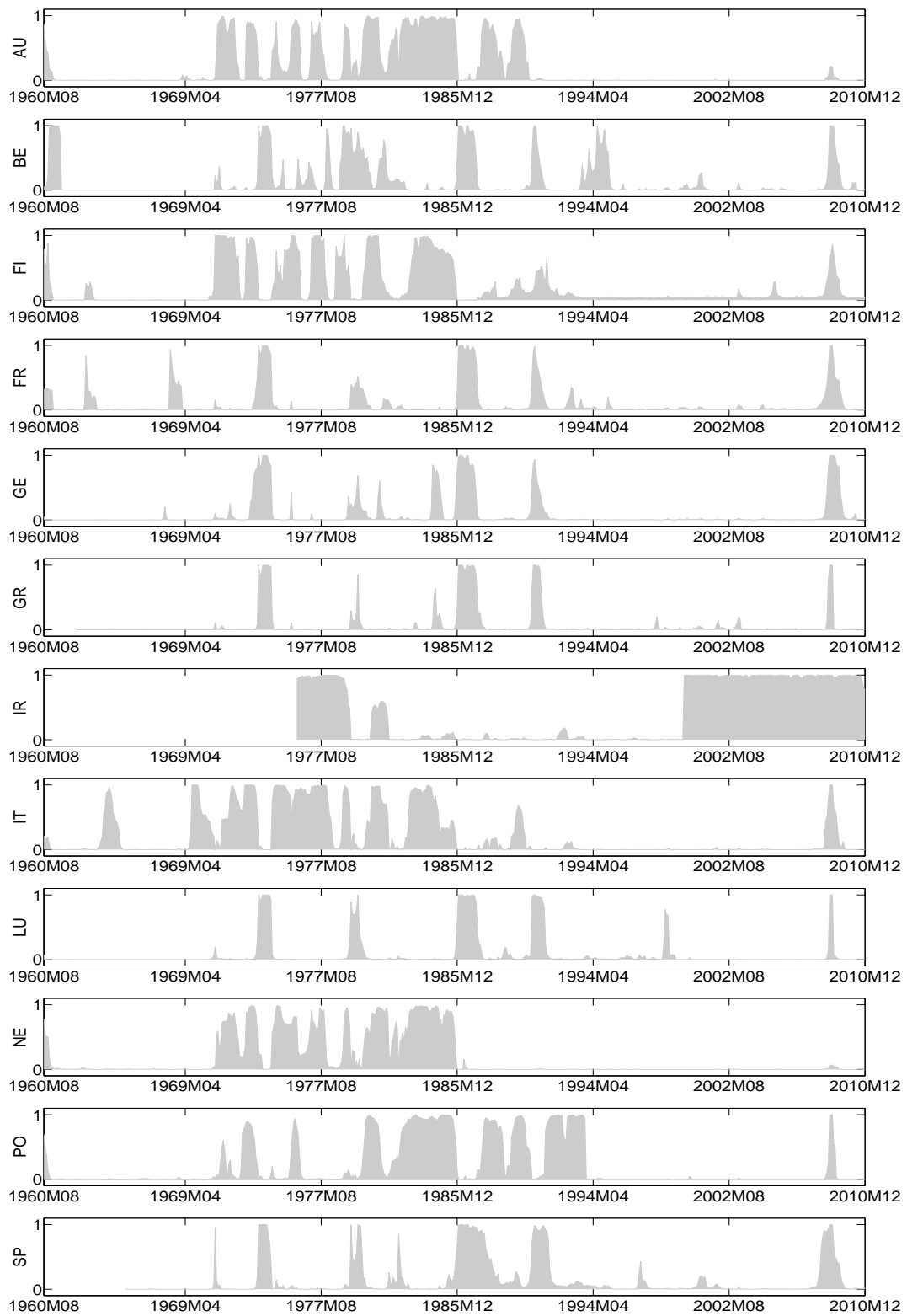
Figure 6: First regime (recession) smoothed probabilities for the Markov-switching processes $s_{i,t}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$.
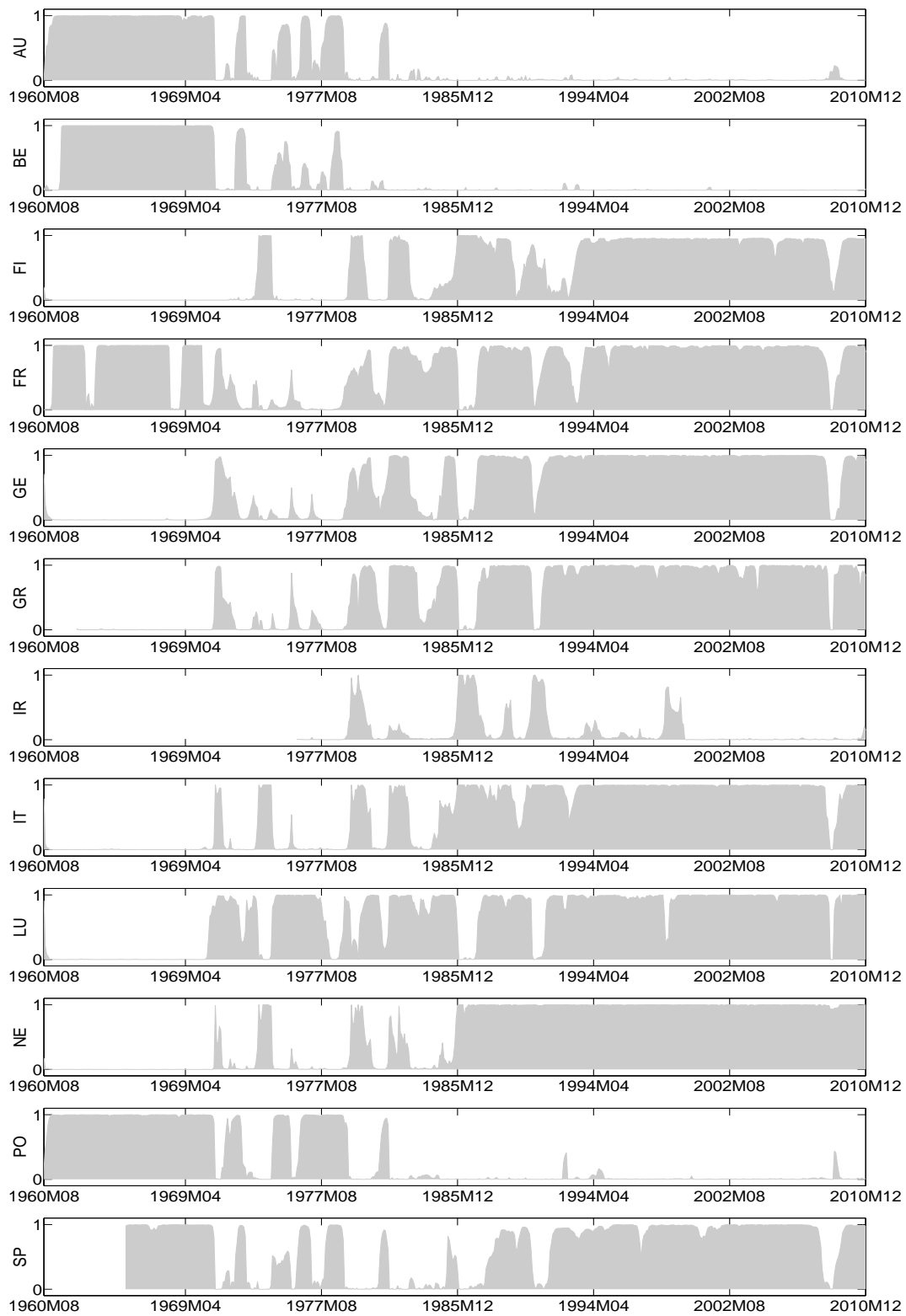
Figure 7: Second regime (moderate expansion) smoothed probabilities for the Markov-switching processes $s_{i,t}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$.
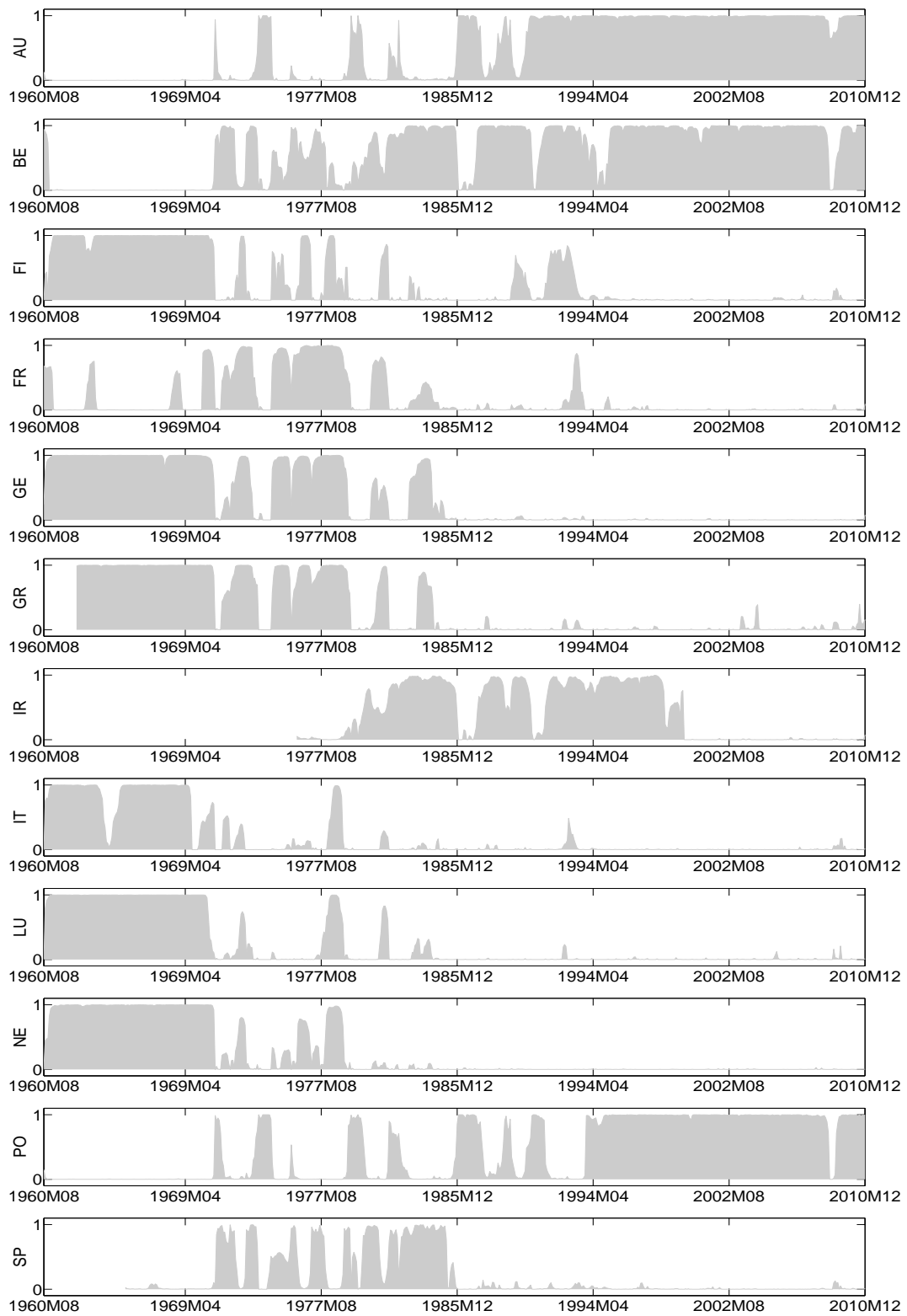
Figure 8: Third regime (strong expansion) smoothed probabilities for the Markov-switching processes $s_{i,t}$, $i = 1, \ldots, N$ and $t = 1, \ldots, T$.

Canova, F. and Marrinan, J. (1998). Sources and propagation of international cycles: common shocks or transmission? *Journal of International Economics*, 42:133–167.

Carter, C. K. and Kohn, R. (1994). On Gibbs Sampling for State Space Models. *Biometrika*, 81(3):541–553.

Celeux, G. (1998). Bayesian Inference for Mixture: The Label Switching Problem. Preprint INRIA.

Clements, M. P. and Krolzig, H. M. (1998). A comparison of the forecast performances of Markov-switching and threshold autoregressive models of US GNP. *Econometrics Journal*, 1:C47–C75.

Dees, S., Di Mauro, F., Pesaran, M., and Smith, L. (2007). Exploring the international linkages of the Euro area: a global VAR analysis. *Journal of Applied Econometrics*, 22:1–38.

Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2000). The generalized dynamic factor model: identification and estimation. *The Review of Economics and Statistics*, 82:540–554.

Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2001). Coincident and leading indicators for the euro area. *The Economic Journal*, 111:C62–C85.

Frankel, J. and Rose, A. (1998). The Endogeneity of the Optimum Currency Area Criterion. *The Economic Journal*, 108:1009–1025.

Frühwirth-Schnatter, S. (2001). Markov Chain Monte Carlo Estimation of Classical and Dynamic Switching and Mixture Models. *Journal of the American Statistical Association*, 96(453):194–209.

Frühwirth-Schnatter, S. (2006). *Mixture and Markov-swithing Models*. Springer, New York.

Geweke, J. (1992). Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments. In Bernardo, J. M., Berger, J. O., Dawid, A. P., and Smith, A. F. M., editors, *Bayesian Statistics 4*, pages 169–193. Oxford University Press, Oxford.

Goldfeld, S. M. and Quandt, R. E. (1973). A Markov Model for Switching Regression. *Journal of Econometrics*, 1:3–16.

Gregory, A., Head, A., and Raynauld, J. (1997). Measuring world business cycles. *International Economic Review*, 38:677–701.

Hallin, M. and Liska, R. (2008). Dynamic Factors in the Presence of Block Structure. Economics Working Papers ECO2008/22, European University Institute.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, 57:357–384.

Hess, G. D. and Shin, K. (1997). International and Intranational Business Cycles. *Oxford Review of Economic Policy*, 13:93–109.

Hess, G. D. and Shin, K. (1998). Intranational business cycles in the United States. *Journal of International Economics*, 44:289–313.

Imbs, J. (2004). Trade, finance, specialization and synchronization. *Review of Economics and Statistics*, 86:723–734.

Kaufmann, S. (2010). Dating and forecasting turning points by bayesian clustering with dynamic structure: A suggestion with an application to austrian data. *Journal of Applied Econometrics*, 25:309–344.

Kim, C. J. and Murray, C. J. (2002). Permanent and Transitory Components of Recessions. *Empirical Economics*, 27:163–183.

Kim, C. J. and Nelson, C. R. (1999). Has the U.S. economy become more stable? a Bayesian approach based on a Markov-switching model of the business cycle. *Review of Economics and Economic Statistics*, 81:608–616.

Kim, C. J. and Piger, J. (2000). Common stochastic trends, common cycles, and asymmetry in economic fluctuations. Working paper, n. 681, International Finance Division, Federal Reserve Board, Semptember 2000.

Kose, M., Otrok, C., and Whiteman, C. (2003). International business cycles: world, region, and country-specific factors. *American Economic Review*, 93:1216–1239.

Kose, M., Otrok, C., and Whiteman, C. (2008). Global business cycles: convergence or decoupling? NBER Working paper 14292.

Krolzig, H.-M. (1997). *Markov Switching Vector Autoregressions. Modelling, Statistical Inference and Application to Business Cycle Analysis*. Springer, Berlin.

Krolzig, H.-M. (2000). Predicting Markov-Switching Vector Autoregressive Processes. Nuffield College Economics Working Papers, 2000-WP31.

Lumsdaine, R. and Prasad, E. (2003). Identifying the common component of international economic fluctuations: a new approach. *Economic Journal*, 113:101–127.

Monfort, A., Renne, J., R., R., and Vitale, G. (2003). Is economic activity in the G7 synchronized? common shocks versus spillover effects. CEPR Discussion Paper No. 4119, Centre for Economic Policy Research, London.

Pesaran, M., Schuermann, T., and Weiner, S. (2004). Modelling regional interdependencies using a global error correcting macroeconometric model. *Journal of Business and Economic Statistics*, 22:129–162.

Potter, S. M. (1995). A Nonlinear Approach to U.S. GNP. *Journal of Applied Econometrics*, 10:109–125.

Robert, C. P. and Casella, G. (1999). *Monte Carlo Statistical Methods*. Springer Verlag, New York.

Shephard, N. (1994). Partial non-Gaussian state space. *Biometrika*, 81:115–131.

Stock, J. H. and Watson, M. W. (1991). A probability model of the coincident economic indicators. In Lahiri K., M. G., editor, *Leading Economic Indicators*. Cambridge University Press, New York.

Tong, H. (1983). *Threshold Models in Non-Linear Time-Series Models*. Springer-Verlag, New York.