Bayesian Stochastic Frontier Analysis of Economic Growth in the EU

Kamil Makieła

Kielce University of Technology

Author Note

Kamil Makieła, Chair of Economics and Management, Kielce University of Technology.

The research is co-founded by the European Union from the European Social Fund (ESF), project ”INWENCJA – potencjał młodych naukowców oraz transfer wiedzy i innowacji wsparciem dla kluczowych dziedzin świętokrzyskiej gospodarki”, identification number: WND-POKL.08.02.01-26-020/11.

Correspondence concerning this article should be addressed to Kamil Makieła, Chair of Economics and Management, Faculty of Management and Computer Modeling, Kielce University of Technology, al. Tysiąclecia Państwa Polskiego 7, 25-314 Kielce, Poland. Email: k.makiela@tu.kielce.pl

Abstract

The paper considers Bayesian approach to productivity analysis of the EU. Bayesian Stochastic Frontier models and structural decomposition of output growth are used to obtain the components of output growth. This allows us to explore the impact of capital accumulation, labor growth, technical progress and technical efficiency change on economic development of the EU. Since estimates of the growth components are conditioned upon model parameterization and the underlying assumptions, a number of possible specifications are proposed. Then, the optimal model is chosen based on the highest marginal data density criterion.

Keywords: stochastic frontier models, Bayesian inference, productivity analysis.

JEL classification: C11, C23, O47, O52.

Bayesian Stochastic Frontier Analysis of Economic Growth in the EU

Production frontier analysis was first proposed by Koopmans (1951) and Debreu (1951) who formed the theoretical basis later used by Farrell (1957) in his pioneering work on efficiency analysis of the US agriculture industry. In the context of frontier analysis one can also compare the entire economies as producing a mutually comparable product (e.g., GDP) using a set of production factors (e.g., physical capital and labor) under a common technology; see, e.g., Fried et al. (2008) for a lengthy list of examples. In such pre-defined model economic growth, i.e., increase in GDP from one period to another, can be caused by i) accumulation of production factors (IC), ii) increase in technical efficiency (EC) or iii) technical progress (TC). This concept was first utilized in the context of frontier analysis by Färe et al. (1994), who used Data Envelopment Analysis (DEA) to analyze economic growth of selected countries. Later, Koop et al. (1999) proposed a Bayesian approach to derive components of output growth. More recently, researchers' attention has also turned to investigating the impact of capital accumulation on economic growth among the EU member states (see, e.g., Salinas-Jiménez et al., 2006). Thus, one may want to decompose the IC component in order to analyze impact of each production factor separately. Furthermore, many researchers seem to prefer DEA as a tool for such macro-scale analysis. This is because, as proponents of DEA argue, being a nonparametric approach DEA does not require imposing any structure on the production frontier. Bayesian estimation, however, has several advantages over DEA. First, it allows us to obtain exact small sample results, which is of particular importance in small macroeconomic datasets. Second, parametric approach is less affected by outlying observations and any nuisance in the data (Fried et al., 2008). Third, though the problem of proper model parameterization remains, one can choose the best model based on a specific, pre-defined criterion.

The aim of this work is to use Bayesian stochastic frontier models in order to trace changes in economic growth patterns among the EU countries[1] in 2000-2010. The output decomposition methodology is based on Koop et al. (1999) and has been extended by additional decomposition of IC component to trace changes of capital and labor contribution to economic growth in the EU. Moreover, since I consider the total of 14 plausible model specifications I set what I think is an intuitive Bayesian criterion for choosing the optimal

[1] I use aggregated data for EU12 countries.

model – the highest marginal data density.

The data used in the analysis come from AMECO database, supervised by DG ECFIN of the European Commission. These are GDP in mld PPS in 2000 constant prices, net capital stock in mld PPS in 2000 constant prices and total number of hours worked annually in a given country (in thousands). The paper is structured as follows.[2] Section 2 outlines the Bayesian stochastic frontier models used in this research. Section 3 provides details on structural decomposition methodology while section 4 briefly describes the method used for choosing the optimal model. Section 5 provides results analysis and section 5 concludes.

Bayesian stochastic frontier models

Let Y_{it}, K_{it} and L_{it} be the levels of production, capital and labor respectively in i-th country (i = 1, …, N) in t-th period (t = 1, …, T), and the lowercase letters y_{it}, k_{it} and l_{it} indicate their natural logs. The general model takes the following form

$$y_{it} = h\,(k_{it}, l_{it}; \beta) + v_{it} - u_{it} \tag{1}$$

where $h(.)$ is the log form of a production function, v_{it} is the independently normally distributed variable with zero mean and an unknown precision σ^{-2}, and u_{it} defines inefficiency term in a way that technical efficiency is r_{it}=exp(-u_{it}) where $0<r_{it}\leq 1$, and $r_{it}=1$ is maximum efficiency. The parametric specifications are

- Cobb-Douglas production function (labeled CD hereafter)

$$h(k_{it}, l_{it}; \beta) = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} \tag{2}$$

- Cobb-Douglas production function with time trend (CDt hereafter)

$$h(k_{it}, l_{it}; \beta) = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 t \tag{3}$$

- Cobb-Douglas production function with linear trend in each parameter of the function (CD-LT hereafter)

$$h(k_{it}, l_{it}; \beta_t) = \beta_{t0} + \beta_{t1} k_{it} + \beta_{t2} l_{it}$$

where $\beta_{ta} = \dot{\beta}_a + t\ddot{\beta}_a$ (a=0,1,2). This equation can be rearranged as

$$h(k_{it}, l_{it}; \beta) = \dot{\beta}_0 + \dot{\beta}_1 k_{it} + \dot{\beta}_2 l_{it} + t(\ddot{\beta}_0 + \ddot{\beta}_1 k_{it} + \ddot{\beta}_2 l_{it}) \tag{4}$$

- Translogarithmic (translog hereafter) production function (labeled TR)

$$h(k_{it}, l_{it}; \beta) = \beta_0 + \beta_1 k_{it} + \beta_2 l_{it} + \beta_3 k_{it}^{\ 2} + \beta_4 l_{it}^{\ 2} + \beta_5 k_{it} l_{it} \tag{5}$$

- Translog production function with time trend (TRt hereafter)

[2] Structure of this paper is based on the author's previous work "Dekompozycja strukturalna wzrostu gospodarczego z wykorzystaniem bayesowskich modeli granicznych na przykładzie krajów EU15" written in Polish.

$$h(k_{it},l_{it};\beta)=\beta_0+\beta_1 k_{it}+\beta_2 l_{it}+\beta_3 k_{it}^{\ 2}+\beta_4 l_{it}^{\ 2}+\beta_5 k_{it} l_{it}+\beta_6 t \quad (6)$$

- Translog production function with linear trend in each parameter (TR-LT)

$$h(k_{it},l_{it};\beta_t)=\beta_{t0}+\beta_{t1} k_{it}+\beta_{t2} l_{it}+\beta_{t3} k_{it}^{\ 2}+\beta_{t4} l_{it}^{\ 2}+\beta_{t5} k_{it} l_{it}$$

where $\beta_{ta}=\dot{\beta}_a+t\ddot{\beta}_a$ (a=0,…,5). The equation can be rearranged as

$$\begin{aligned} h(k_{it},l_{it};\beta)&=\dot{\beta}_0+\dot{\beta}_1 k_{it}+\dot{\beta}_2 l_{it}+\dot{\beta}_3 k_{it}^{\ 2}+\dot{\beta}_4 l_{it}^{\ 2}+\dot{\beta}_5 k_{it} l_{it} \\ &+t(\ddot{\beta}_0+\ddot{\beta}_1 k_{it}+\ddot{\beta}_2 l_{it}+\ddot{\beta}_3 k_{it}^{\ 2}+\ddot{\beta}_4 l_{it}^{\ 2}+\ddot{\beta}_5 k_{it} l_{it}) \end{aligned} \quad (7)$$

- Translog production function with quadratic trend in each parameter (TR-QT):

$$h(k_{it},l_{it};\beta_t)=\beta_{t0}+\beta_{t1} k_{it}+\beta_{t2} l_{it}+\beta_{t3} k_{it}^{\ 2}+\beta_{t4} l_{it}^{\ 2}+\beta_{t5} k_{it} l_{it}$$

where $\beta_{ta}=\dot{\beta}_a+t\ddot{\beta}_a+t^2\dddot{\beta}_a$ (a=0,…,5). Like in TR-LT this can be rearranged as follows

$$\begin{aligned} h(k_{it},l_{it};\beta)&=\dot{\beta}_0+\dot{\beta}_1 k_{it}+\dot{\beta}_2 l_{it}+\dot{\beta}_3 k_{it}^{\ 2}+\dot{\beta}_4 l_{it}^{\ 2}+\dot{\beta}_5 k_{it} l_{it} \\ &+t(\ddot{\beta}_0+\ddot{\beta}_1 k_{it}+\ddot{\beta}_2 l_{it}+\ddot{\beta}_3 k_{it}^{\ 2}+\ddot{\beta}_4 l_{it}^{\ 2}+\ddot{\beta}_5 k_{it} l_{it}) \\ &+t^2(\dddot{\beta}_0+\dddot{\beta}_1 k_{it}+\dddot{\beta}_2 l_{it}+\dddot{\beta}_3 k_{it}^{\ 2}+\dddot{\beta}_4 l_{it}^{\ 2}+\dddot{\beta}_5 k_{it} l_{it}) \end{aligned} \quad (8)$$

Equations (2-8) can be summarized as $h(k_{it},l_{it};\beta)=x'_{it}\beta$ where vector x_{it} is the element of **X** that contains the list of arguments appropriate to the given production function in (2-8). This work also considers two most commonly used specifications for prior distribution of the inefficiency term, i.e., exponential and half-normal (Greene, 2008). As a result this gives us two model “classes” – normal-exponential (labeled NExp hereafter) and normal-half-normal (NHN hereafter) – with seven possible parameterizations of the production function per model class. Hence, the total number of models considered amounts to 14. The full Bayesian NExp-class model is[3]

$$\begin{aligned} &\prod_{i=1}^{N}\prod_{t=1}^{T} f_N(y_{it}\,|\,h(k_{it},l_{it},\beta)-u_{it},\sigma^2)\, f_N(\beta\,|\,b,C^{-1})\cdot \\ &f_G(\sigma^{-2}\,|\,0.5n_0,0.5a_0)\, f_G(\lambda^{-1}\,|\,1,-\ln(r_0))\, f_G(u_{it}\,|\,1,\lambda^{-1}) \end{aligned} \quad (9)$$

where $f_N(.\,|\,w,Z)$ is a normal density function with w mean and Z covariance matrix, $f_G(.\,|\,w,z)$ is a gamma density function with w/z mean and w/z^2 variance. I set $n_0=a_0=10^{-6}$ which leads to a quite flat distribution for σ^{-2} with mean 1 and variance $2\cdot 10^6$. The r_0 parameter, prior median efficiency, is set as 0.75, giving equal prior chances that technical efficiency of a given country is either greater or smaller than 75%. Other values were also

[3] The model structure is similar to Koop et al. (1999), the difference being prior on β.

considered (0.65 and 0.875). This, however, had virtually no impact on the results.[4] To allow for cross-model comparability b and C^{-1} parameters have been calibrated so that the prior on β shares the following properties in all models: i) average elasticities of capital and labor have 0.5 prior means with 0.2 prior standard deviation[5], ii) neutral technical change has prior mean of 0.02 and 0.01 prior standard deviation. Economic regularity conditions (non-negative factor elasticities) are imposed through inequalities appropriate to the given parametric specification. The model is too complex to analytically acquire marginal posterior distributions of its parameters. We can, however, draw from their conditional posterior distributions, which are

$$
\begin{aligned}
&\mathrm{p}(\beta \mid y, X, u, \lambda^{-1}, \sigma^{-2}) \propto f_N^J(\beta \mid C_*^{-1}[Cb + \sigma^{-2} X'(y+u)], C_*^{-1}) \\
&\mathrm{p}(\sigma^{-2} \mid y, X, u, \lambda^{-1}, \beta) \\
&\propto f_G(\sigma^{-2} \mid \frac{n_0 + NT}{2}, 0.5[a_0 + (y + u - h(X;\beta))'(y_t + u_t - h(X;\beta))]) \\
&\mathrm{p}(u \mid y, X, \lambda^{-1}, \sigma^{-2}, \beta) \\
&\propto f_N{}^{NT}(u \mid h(X,\beta) - y - \sigma^2 \lambda^{-1}, \sigma^2 \cdot I_{NT}) I(u \in R_+^{NT}) \\
&\mathrm{p}(\lambda^{-1} \mid y, X, u, \sigma^{-2}, \beta) \propto f_G(\lambda^{-1} \mid NT + 1, \sum_{n=1}^{N} \sum_{t=1}^{T} u_{it} - \ln(r_0))
\end{aligned}
\qquad (10)
$$

where $C_*^{-1} = (C + \sigma^{-2} X'X)^{-1}$ and J is the number of elements in β. Based on the formulas above and with a moderate numerical effort one can approximate characteristics of the marginal distributions using Gibbs sampler. The full Bayesian NHN-class model used in the study is

$$
\begin{aligned}
&\prod_{i=1}^{N} \prod_{t=1}^{T} f_N(y_{it} \mid h(k_{it}, l_{it}, \beta) - u_{it}, \sigma^2) f_N(\beta \mid b, C^{-1}) \cdot \\
&f_G(\sigma^{-2} \mid 0.5 n_0, 0.5 a_0) f_G(\omega^{-2} \mid 5, 10 \ln^2(r_0)) f_N(u_{it} \mid 0, \omega^2)
\end{aligned}
\qquad (11)
$$

where $n_0 = a_0 = 10^{-6}$ and $r_0 = 0.75$ and the prior on ω^{-2} is as proposed in van den Broeck et al. (1994). Like in the case of NExp, this model is also very complex and thus the characteristics of marginal distributions must be approximated numerically, e.g., using Gibbs sampler and the known conditionals. For β and σ^2 the conditionals do not change in relation to NExp model. Conditionals for the remaining parameters are

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[4] It only (slightly) influenced the average level of efficiency in the sample, mostly when dealing with Cobb-Douglas models. Efficiency scores in translog models were more robust in this regard. Also, efficiency scores in NExp models were slightly more influenced by the prior than NHN models. Again, this was quite evident in Cobb-Douglas production specifications, but not so much in case of translog specifications.

[5] Prior standard deviations for elasticities (at their means) are slightly higher in models with translog production function. This, however, is bound to happen if the covariance matrix is to be invertible.

$$\mathrm{p}(\omega^{-2} \mid y, X, u, \sigma^{-2}, \beta) \propto f_G(\omega^{-2} \mid 0.5NT + 5, 0.5 \cdot \sum_{n=1}^{N}\sum_{t=1}^{T} u_{it} + 10\ln^2(r_0))$$
$$\mathrm{p}(u \mid y, X, \omega^{-2}, \sigma^{-2}, \beta) \propto f_N{}^{NT}(u \mid \frac{\omega(h(X;\beta) - y)}{\omega^2 + \sigma^2}, \frac{\omega^2\sigma^2}{\omega^2 + \sigma^2}) I(u \in R_+^{NT}) \qquad (12)$$

Based on the above, 14 models where estimated using Gibbs sampler coded in MATLAB. Five hundred thousand draws where taken, discarding initial 100 000 (burn-in process). Convergence of the chain during each simulation to its limiting stationary distribution was monitored using both sequential plots and cusum paths (Yu & Mykland, 1998). Sequential plots were primarily used to asses if the burn-in stage is long enough, while cusum plots were used to analyze the sampler's mixing speed. All simulations stabilized way before the end of their burn-in periods and the sampler's mixing speed was either very good[6], or at least satisfactory in the case of two most complex normal-half-normal models – TR-LT and TR-QT (see appendix A). Hence both, the number of burn-in cycles and the number of accepted draws could have been smaller if we were to base our analysis on standard statistics such as posterior means and posterior standard deviations. In this case, however, long runs were necessary to acquire precise estimates of the marginal data densities for all models, especially those where sampler's mixing speed was lower.

Structural decomposition of output growth

The difference in the log of GDP between two corresponding periods *t* and *t+1* can be written as (Koop et al., 1999)

$$\Delta y = 0.5(x_{i,t+1} + x_{it})'(\beta_{t+1} - \beta_t) + 0.5(\beta_{t+1} + \beta_t)'(x_{i,t+1} - x_{it}) + (u_{it} - u_{i,t+1}) \qquad (13)$$

where the first term reflects output change due to technical progress (or regress), the second is due to change in production factors, and the third reflects changes to technical efficiency. This allows us to derive three main components of output growth

$$IC_{i,t+1} = \exp\left[0.5(\beta_{t+1} + \beta_t)'(x_{i,t+1} - x_{it})\right] \qquad (14)$$

$$TC_{i,t+1} = \exp\left[0.5(x_{i,t+1} + x_{it})'(\beta_{t+1} - \beta_t)\right] \qquad (15)$$

$$EC_{i,t+1} = \exp(u_{it} - u_{i,t+1}) \qquad (16)$$

and the joint impact of TC and EC as

$$PC_{i,t+1} = EC_{i,t+1} \times TC_{i,t+1} \qquad (17)$$

which is a standard Malmquist productivity index (Caves et al., 1982). Equations (13-17) summarize decomposition methodology introduced by G. Koop, J. Osiewalski and M.F.J.

[6] That means highly oscillatory cusum paths with low excursion and hardly any difference in comparison to their benchmark paths.

Steel (1999) in the context of Bayesian stochastic frontier models. In this work, however, I want to investigate the contribution of each factor to economic growth separately. Thus, for a two-factor production function equation (14) is broken down into

$$\begin{aligned} IC_{i,t+1} &= \exp\left[\Delta \mathrm{f}1_{i,t+1} \cdot El.\mathrm{f}1(0.5(x_{i,t+1} + x_{it}))\right] \times \\ &\exp\left[\Delta \mathrm{f}2_{i,t+1} \cdot El.\mathrm{f}2(0.5(x_{i,t+1} + x_{it}))\right] = IC.\mathrm{f}1_{i,t+1} \times IC.\mathrm{f}2_{i,t+1} \end{aligned} \tag{18}$$

where f1 and f2 are the production factors (e.g., capital and labor). To sum up, the change in GDP level between period *t* and *t+1* ($OC_{i,t+1}$) can be summarized as

$$OC_{i,t+1} = ICk_{i,t+1} \times ICl_{i,t+1} \times TC_{i,t+1} \times EC_{i,t+1} \tag{19}$$

where components on the right side of (19) reflect the impact of capital change, labor change, technical progress and efficiency change on economic growth respectively. To make the interpretation more intuitive (and comparable to Koop et al. 1999), indicators from (14-19) are given as percentage changes to the previous year. Thus, a simple transformation Δ%=100%(δ-1) is used, where δ is the initial level of an indicator from (14-19).

It should be noted that the model choice has a profound impact on the structure and the detail of the output growth decomposition. That is why when choosing the optimal model a research should also consider the following. CD model does not allow for technical progress (frontier shifts) and since factors' elasticities are constant among all observations, changes in the IC component can be caused only by changes in factors' input levels. CDt model does allow for a technical change which, however, is constant not only through time but across countries as well. CD-LT model allows the technology to change, but only over time and in a linear fashion. TR model allows us to consider technical change more flexibly through factors' elasticities, which can vary over time and across countries. It is not possible, however, to distil the effect of technical change from the impact of input change component. TRt model solves this problem only partially because we face the same issue as in CDt model – unrealistic assumption regarding technical change. Full-scale decomposition can be obtained with TR-LT model. Introducing linear trend into each parameter of the production function allows the technical change to impact each country differently over time (though in a linear fashion). TR-QT model further loosens the restrictions on how technical change can impact a country's growth. In doing so, it allows us to investigate changes in technical progress contribution to a given country's economic growth over time.

Choosing the optimal model

One of key aspects of Bayesian analysis is to compute marginal data density given as

$$p(y \mid M_k) = \int p(\Theta_k \mid M_k)\, p(y \mid \Theta_k, M_k)\, d\Theta_k \tag{20}$$

where Θ donates a set of parameters of k-th model $M=\{M_1,\ldots,M_k,\ldots\}$ and y is the data vector. Since this paper uses Gibbs sampler for model estimation (which is an MCMC class algorithm) we can use the following harmonic average to approximate the marginal data density of each model (Newton & Raftery, 1994):

$$p(y \mid M_k)^{-1} = \frac{1}{R}\sum_{r=1}^{R} p(\Theta^{(r)}{}_k \mid M_k)^{-1} \qquad (21)$$

where $\Theta^{(r)}{}_k$ are MCMC draws given k-th model, and R is the number of accepted draws.

[Table 1 here]

Table 1 presents the main estimation results for all 14 models. The second column shows results for marginal data density for each model (as tenth logarithms). Having this we can easily construct the Bayes factor and conclude that i) normal-half-normal models are more preferred to normal-exponential models and that ii) Cobb-Douglas production function is the least favored parametric specification given the data. The normal-half-normal model with quadratic trend in parameters has indisputably the highest marginal data density of all models considered. Thus, the next section discusses results based on this parametric specification.

Results analysis

On average the UE member states performed poorly over the last decade, especially the EU15 region. Only a handful of EU15 economies increased their productivity. The main driver of economic growth was the increase in inputs, and mainly capital accumulation. The EU12 region was also under-performing in terms of productivity change. However, in this case the impact of capital accumulation on GDP growth was very strong and amounted to an average of 3.81% (0.26%)[7] annually. Moreover, EU12 had the highest returns to scale ratio – 1.03 (0.02). This explains why the EU12 zone more than doubled the growth rate of the “old Union” and provides evidence in favor of an ongoing convergence process in the EU. The convergence seems to have two sources, high capital accumulation and (also high) increasing returns to scale of the EU12 region.

As far as development of the EU15 member states is concerned, capital accumulation has had the highest impact on Ireland 3.13% (0.21%) and Luxembourg 3.46% (0.39%). The productivity component has been a crucial element of economic growth in Sweden 0.67% (0.13%) and Finland 0.64% (0.13%). Further decomposition of the productivity component reveals that in both cases it was the increase in technical efficiency that contributed to

[7] All point estimates are posteriori means, posteriori standard deviations are given in brackets.

economic growth in those countries. Technical change component was the highest in Germany 0.46% (0.53%) and France 0.26% (0.58%). However, high posterior standard deviations (in relation to their means) indicate that one should be careful with drawing any conclusions.

[Table 2 here]

Around 2008-09 all EU economies have suffered from a drastic technical efficiency plunge, not doubt due to the world-wide financial crisis. Irish economy was among the first to be affected. The country's technical efficiency went down by 4.6% (1.16%) in 2007-08 and by 5.52% (1.26%) in 2008-09, which was the third highest one-time efficiency drop in the analyzed period. The two highest drops occurred (in 2008-09) in Finland 6.93% (1.16%) and Luxembourg 6.49% (1.36%). Interestingly, efficiency growth was still the main driver of economic growth in Finland. All EU15 countries apart from Greece have started to recover by the end of 2010. EU12 economies also suffered a drastic one-time drop in technical efficiency by 5.66% (1.86%) but recovered the following year.

[Figure 1 here]

As far as the structure of economic growth is concerned, the worst results come from Greek and Portuguese economies. Greece has turned out to be (on average) the least efficient economy in the sample and was still losing on efficiency at the end of the analyzed period, while most economies were already recovering. Portugal placed itself as the second least efficient economy with an average decline in productivity by 0.88% (0.18%) annually. Estimation results indicate that if it hadn't been for the strong effect of capital accumulation on Portuguese GDP, the country would have been in recession long before the crisis.

[Table 3 here]

The two countries, however, do not share the same dynamics as far as technical progress and technical efficiency change are concerned. Though at the beginning of the analyzed period the Greek economy was doing better than Portuguese, the crisis has had a much bigger impact on it.

[Figure 2 here]

Concluding remarks

Regardless of the model considered, capital accumulation has been the main driver of economic growth in the EU over the past decade. Its impact on economic growth was several times higher than that of technical efficiency change – the second most important component. Labor change component has turned out to be less significant than capital accumulation and even efficiency change. Technical progress has had a marginal impact on economic growth in the EU over the past decade.

Efficiency change (EC) and technical change (TC) components (high posterior standard deviations in respect to their means) were difficult to precisely estimate. One reason for this is that efficiencies are *latent* variables in the models and even though their levels can be estimated fairly precisely, first difference estimates are bound to be highly dispersed. Second, the models were estimated using data covering the world-wide financial (and thus economic) meltdown. This must have had an influence on the precision of the technical change estimates. The 2008-09 downturn has largely wrecked any economic progress achieved up to 2007, so it is hard to expect technical progress to be an important factor of economic growth in that period.

[Figure 3 here]

To sum up, it should be noted that the use of Bayesian stochastic frontier models allowed us to choose optimal model not only based on theoretical guidelines briefly mentioned at the end of section 3 but primarily based on the information in the data. In particular the study indicates that Cobb-Douglas parameterization is too restrictive to be used in such studies and that, at least for this dataset, normal-half-normal models are more preferred to normal-exponential model.

Acknowledgements

The research is co-founded by the European Union from the European Social Fund (ESF), project ”INWENCJA – potencjał młodych naukowców oraz transfer wiedzy i innowacji wsparciem dla kluczowych dziedzin świętokrzyskiej gospodarki”, identification number: WND-POKL.08.02.01-26-020/11.

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Table 1

Main estimation results

	$\log_{10}$	Capital elast.		Labor elast.		RTS		σ		ω^2	
NHN	**p(y|M)**	**E(.)**	**D(.)**	**E(.)**	**D(.)**	**E(.)**	**D(.)**	**E(.)**	**D(.)**	**E(.)**	**D(.)**
TR-QT	102.32	~0.7558		~0.2065		~0.9622		0.0056	0.0056	0.0477	0.0054
TR-LT	89.35	~0.7668		~0.1932		~0.9600		0.0172	0.0166	0.0487	0.0070
TRt	83.72	~0.7652		~0.1887		~0.9539		0.0274	0.0177	0.0452	0.0069
TR	88.42	~0.7674		~0.1864		~0.9538		0.0278	0.0173	0.0451	0.0069
CD-LT	70.66	~0.7748		~0.1563		~0.9311		0.0598	0.0085	0.0453	0.0064
CDt	71.18	0.7783	0.0338	0.1475	0.0326	0.9258	0.0056	0.0581	0.0084	0.0454	0.0064
CD	74.54	0.7709	0.0334	0.1541	0.0324	0.9250	0.0056	0.0586	0.0082	0.0453	0.0064
NEx										λ	
TR-QT	66.72	~0.8013		~0.1403		~0.9417		0.0675	0.0092	0.0918	0.0140
TR-LT	63.52	~0.8032		~0.1373		~0.9405		0.0680	0.0095	0.0935	0.0145
TRt	66.48	~0.8017		~0.1374		~0.9390		0.0666	0.0086	0.0937	0.0135
TR	64.22	~0.7981		~0.1407		~0.9388		0.0659	0.0085	0.0945	0.0134
CD-LT	55.92	~0.7847		~0.1482		~0.9329		0.0764	0.0103	0.1050	0.0152
CDt	55.88	0.7852	0.0274	0.1408	0.0262	0.9261	0.0057	0.0744	0.0100	0.1063	0.0149
CD	57.31	0.7791	0.0268	0.1466	0.0257	0.9256	0.0057	0.0740	0.0097	0.1070	0.0147

Source: author's calculations.

Note. E(.) is the posteriori mean; D(.) is the posteriori standard deviation; ~ indicates an average level.

Table 2

Decomposition results

Country	***av.IC***	***av.IC_k***	***av.IC_l***	***av. PC***	***av. EC***	***av. TC***	***av. OC***	**ΔGDP**
Austria	1.410 *0.079*	1.397 *0.082*	0.013 *0.002*	0.135 *0.137*	0.507 *0.347*	-0.369 *0.335*	1.547 *0.113*	1.54
Belgium	1.506 *0.040*	1.307 *0.076*	0.196 *0.037*	-0.129 *0.111*	0.243 *0.333*	-0.371 *0.332*	1.375 *0.106*	1.37
Denmark	1.109 *0.076*	1.102 *0.077*	0.007 *0.001*	-0.466 *0.129*	0.117 *0.300*	-0.581 *0.307*	0.637 *0.105*	0.64
Finland	1.212 *0.071*	1.170 *0.080*	0.042 *0.009*	0.641 *0.132*	1.202 *0.298*	-0.554 *0.291*	1.861 *0.112*	1.85
France	1.718 *0.140*	1.679 *0.148*	0.038 *0.009*	-0.572 *0.175*	-0.831 *0.634*	0.264 *0.580*	1.136 *0.112*	1.13
Germany	0.866 *0.091*	0.903 *0.082*	-0.037 *0.009*	0.065 *0.143*	-0.390 *0.569*	0.459 *0.534*	0.931 *0.112*	0.93
Greece	1.806 *0.075*	1.678 *0.098*	0.127 *0.023*	0.319 *0.135*	0.785 *0.265*	-0.461 *0.253*	2.132 *0.114*	2.11
Ireland	3.163 *0.203*	3.129 *0.211*	0.033 *0.007*	-0.750 *0.224*	-0.242 *0.303*	-0.509 *0.294*	2.389 *0.114*	2.36
Italy	1.296 *0.079*	1.228 *0.093*	0.067 *0.014*	-0.890 *0.134*	-1.104 *0.323*	0.217 *0.302*	0.394 *0.112*	0.39
Luxembourg	3.825 *0.245*	3.460 *0.397*	0.353 *0.153*	-1.038 *0.256*	-0.949 *0.426*	-0.087 *0.549*	2.747 *0.116*	2.69
Holland	1.361 *0.070*	1.305 *0.080*	0.055 *0.010*	0.007 *0.128*	0.266 *0.329*	-0.257 *0.315*	1.368 *0.109*	1.36
Portugal	1.559 *0.143*	1.609 *0.132*	-0.049 *0.011*	-0.878 *0.178*	-0.042 *0.664*	-0.832 *0.679*	0.667 *0.112*	0.66
Spain	3.298 *0.159*	3.043 *0.207*	0.247 *0.049*	-1.189 *0.187*	-1.196 *0.272*	0.008 *0.235*	2.069 *0.114*	2.05
Sweden	1.366 *0.059*	1.281 *0.074*	0.084 *0.016*	0.674 *0.126*	1.049 *0.326*	-0.371 *0.312*	2.049 *0.114*	2.03
UK	1.750 *0.116*	1.725 *0.120*	0.025 *0.005*	-0.082 *0.147*	-0.256 *0.236*	0.175 *0.251*	1.666 *0.101*	1.63
EU12	3.769 *0.265*	3.811 *0.257*	-0.041 *0.008*	-0.126 *0.279*	0.163 *1.117*	-0.277 *1.140*	3.637 *0.115*	3.57
EU15	~1.596	~1.544	~0.051	~ -0.337	~ -0.444	~0.110	~1.251	~1.24

Source: author's calculations.

Note. "av" indicates posteriori mean of an average change; posteriori standard deviations are in italic; ~ is the average level weighted by EU15 countries' average GDP levels

Table 3

Main economic indicators

	average capital elasticity		average labor elasticity		average RTS		average technical efficiency	
	E(.)	**D(.)**	**E(.)**	**D(.)**	**E(.)**	**D(.)**	**E(.)**	**D(.)**
Austria	0.7500	0.0405	0.1963	0.0394	0.9463	0.0054	0.765	0.015
Belgium	0.7499	0.0403	0.1957	0.0393	0.9456	0.0051	0.946	0.018
Denmark	0.7313	0.0458	0.1900	0.0425	0.9213	0.0060	0.962	0.014
Finland	0.7313	0.0451	0.1888	0.0423	0.9201	0.0055	0.908	0.014
France	0.7907	0.0594	0.2248	0.0528	1.0155	0.0139	0.857	0.026
Germany	0.7878	0.0615	0.2311	0.0556	1.0188	0.0144	0.825	0.023
Greece	0.7496	0.0399	0.2029	0.0393	0.9525	0.0036	0.659	0.009
Ireland	0.7287	0.0436	0.1878	0.0417	0.9165	0.0048	0.869	0.013
Italy	0.7878	0.0518	0.2277	0.0496	1.0154	0.0108	0.839	0.014
Luxembourg	0.6881	0.0666	0.1541	0.0612	0.8421	0.0137	0.948	0.019
Holland	0.7620	0.0425	0.2053	0.0408	0.9673	0.0067	0.916	0.017
Portugal	0.7372	0.0499	0.2056	0.0444	0.9428	0.0115	0.737	0.023
Spain	0.7778	0.0469	0.2235	0.0458	1.0013	0.0087	0.797	0.011
Sweden	0.7507	0.0403	0.1970	0.0393	0.9476	0.0051	0.823	0.015
UK	0.7852	0.0487	0.2300	0.0485	1.0153	0.0098	0.979	0.010
EU12	0.7743	0.0433	0.2440	0.0489	1.0283	0.0159	0.794	0.043
EU15	~0.7780	–	~0.2215	–	~1.0016	–	~0.861	–

Source: author’s calculations.

Note. E(.) is the posteriori mean; D(.) is the posteriori standard deviation; ~ is the average level weighted by EU15 countries’ average GDP levels in the analyzed period.

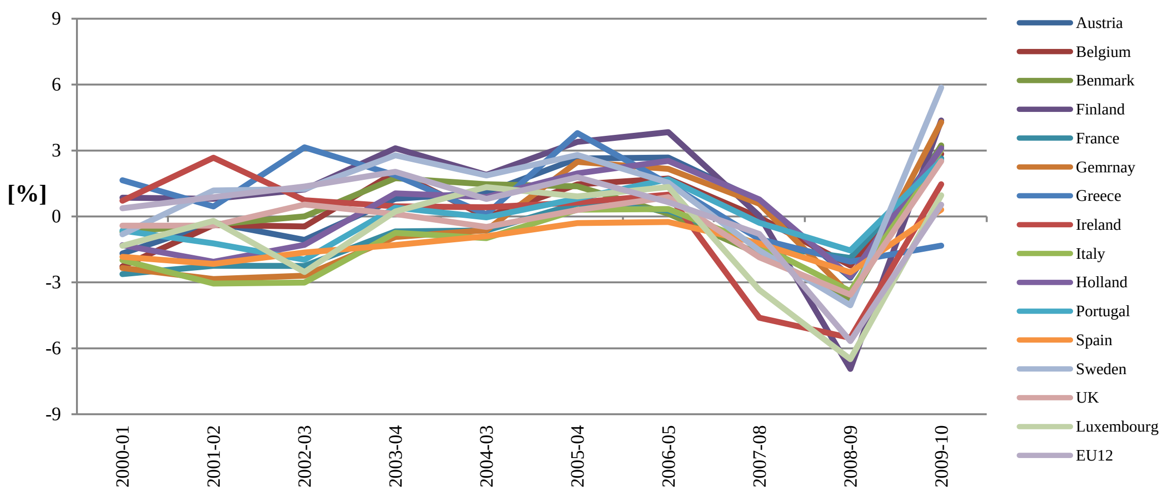

Figure 1. Efficiency changes in the EU member states between 2000 and 2010

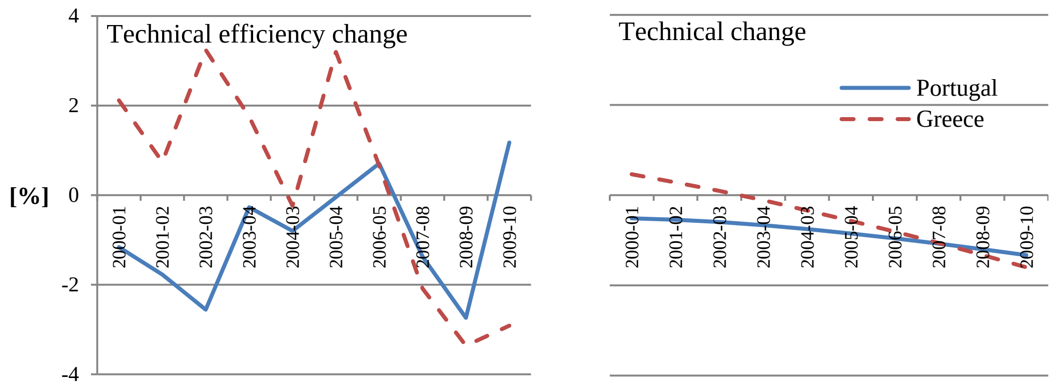

Figure 2. Changes in productivity components; 2000-2010

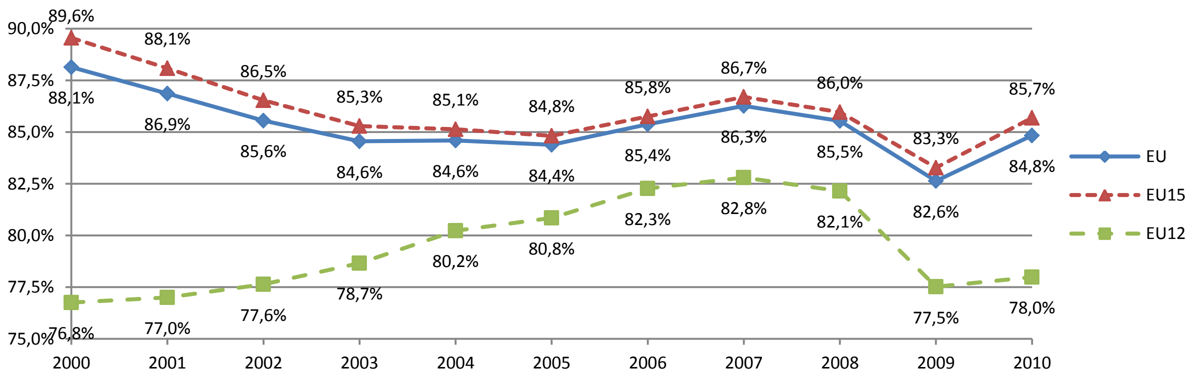

Figure 3. Technical efficiency levels in the EU, EU12 and EU15 in 2000-2010; results for EU and EU15 are calculated as countries average weighted by their average GDP levels in the analyzed period

Appendix A: cusum path plots

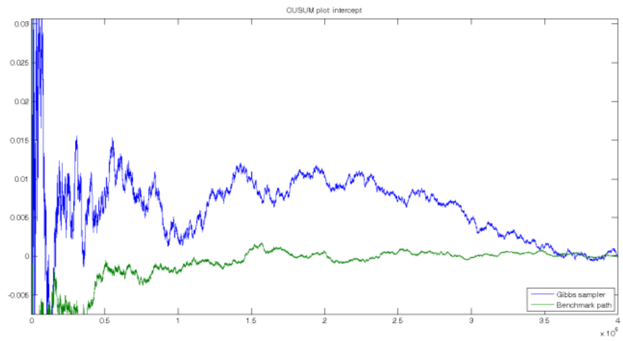

Figure A1. Normal-half-normal CD model

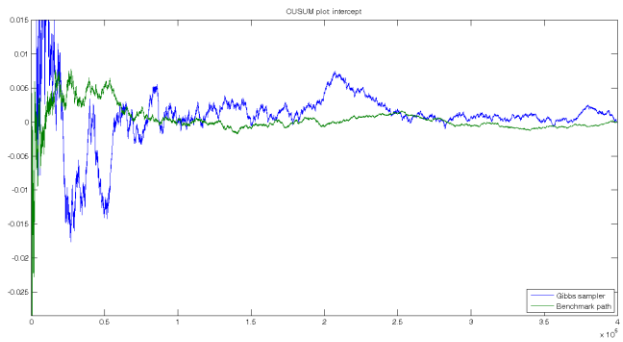

Figure A2. Normal-half-normal CDt model

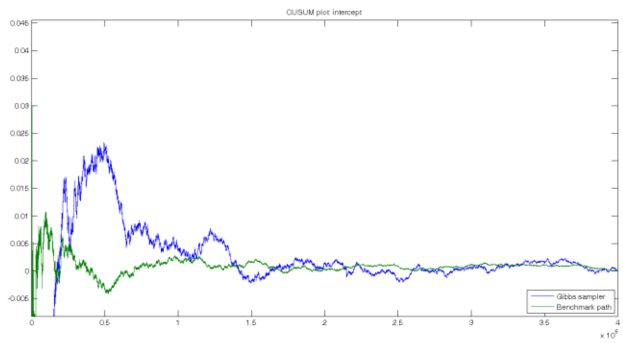

Figure A3. Normal-half-normal CD-LT model

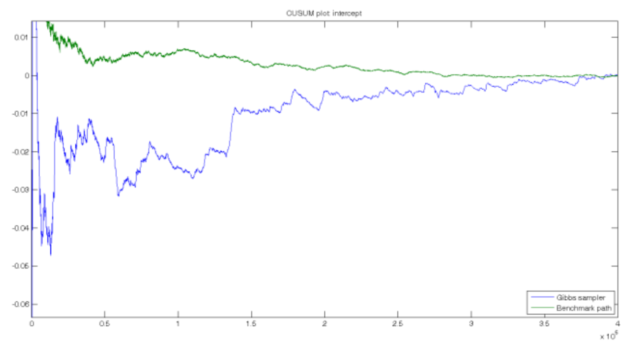

Figure A4. Normal-half-normal TR model

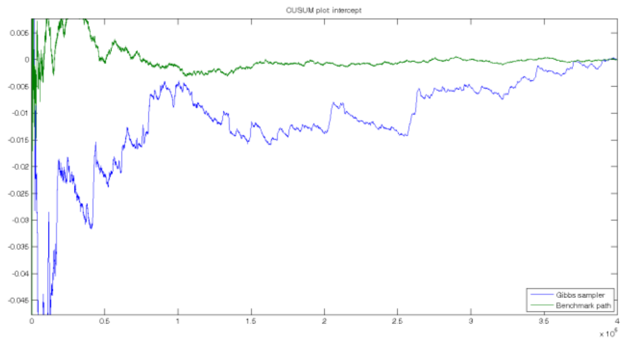

Figure A5. Normal-half-normal TRt model

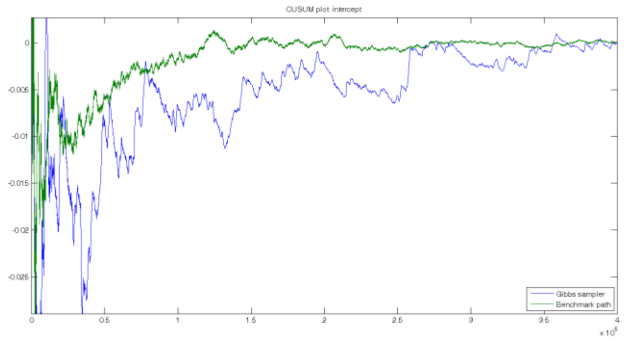

Figure A6. Normal-half-normal TR-LT model

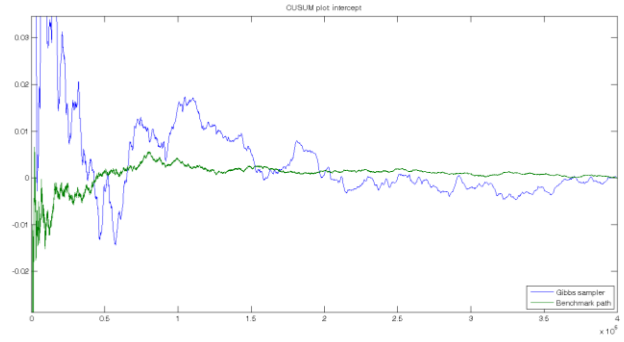

Figure A7. Normal-half-normal TR-QT model

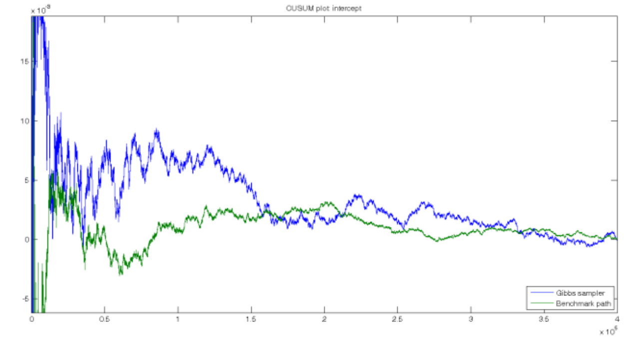

Figure A8. Normal-exponential CD model

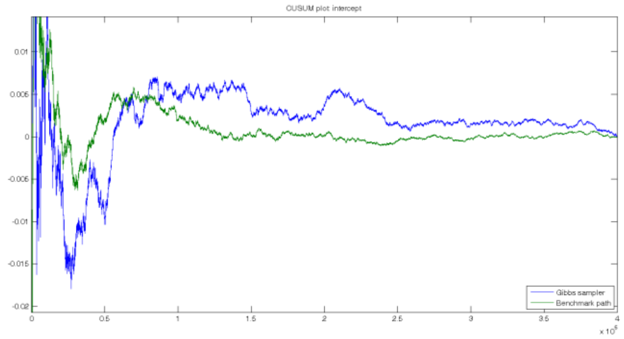

Figure A9. Normal-exponential CDt model

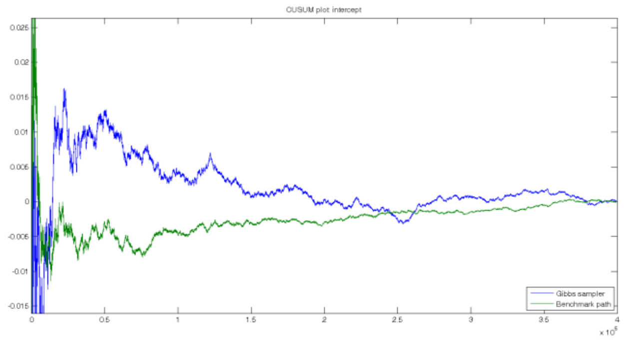

Figure A10. Normal-exponential CD-LT model

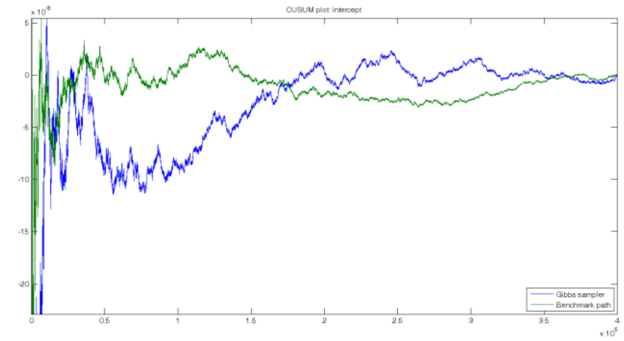

Figure A11. Normal-exponential TR model

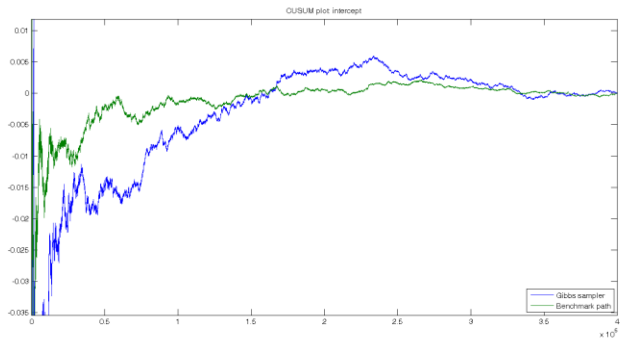

Figure A12. Normal-exponential TRt model

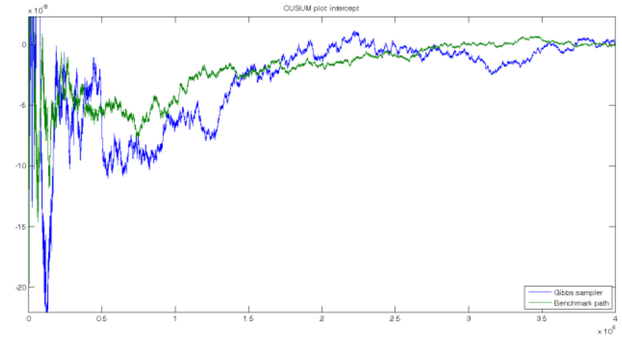

Figure A13. Normal-exponential TR-LT model

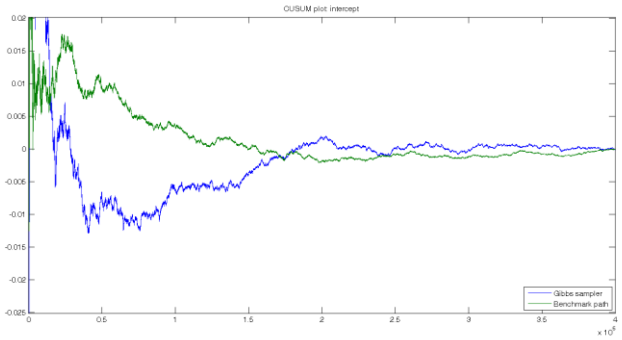

Figure A14. Normal-exponential TR-QT model